

# Speed and Simplicity for Incremental Sequence Computation

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1 [kyleheadley.github.io](https://kyleheadley.github.io)

# What is Incremental Computation?

4 2 5 1 4 9 5 7 8 3 3 6

# What is Incremental Computation?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 9$

# What is Incremental Computation?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 9$

Change Data



4 2 5 1 4 3 5 7 8 3 3 6



# What is Incremental Computation?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 9$

Change Data

Update Result?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 3 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 8$

# What is Incremental Computation?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 9 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 9$

Change Data

Update Result?

$\text{max}( 4 \ 2 \ 5 \ 1 \ 4 \ 3 \ 5 \ 7 \ 8 \ 3 \ 3 \ 6 ) = 8$

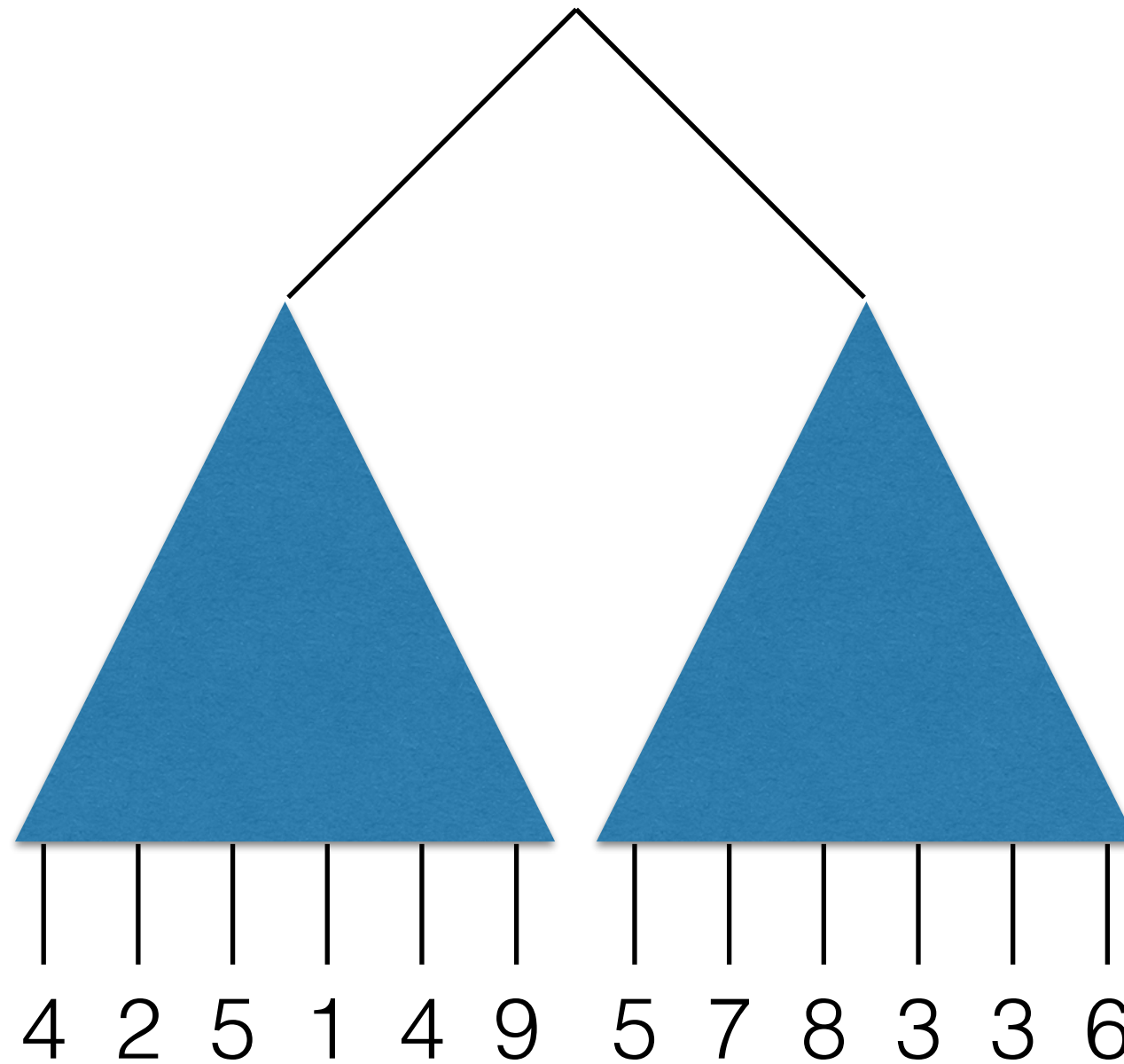
Not incremental: requires additional full scan of data

A computation is incremental if repeating it with a changed input is faster than re-computation

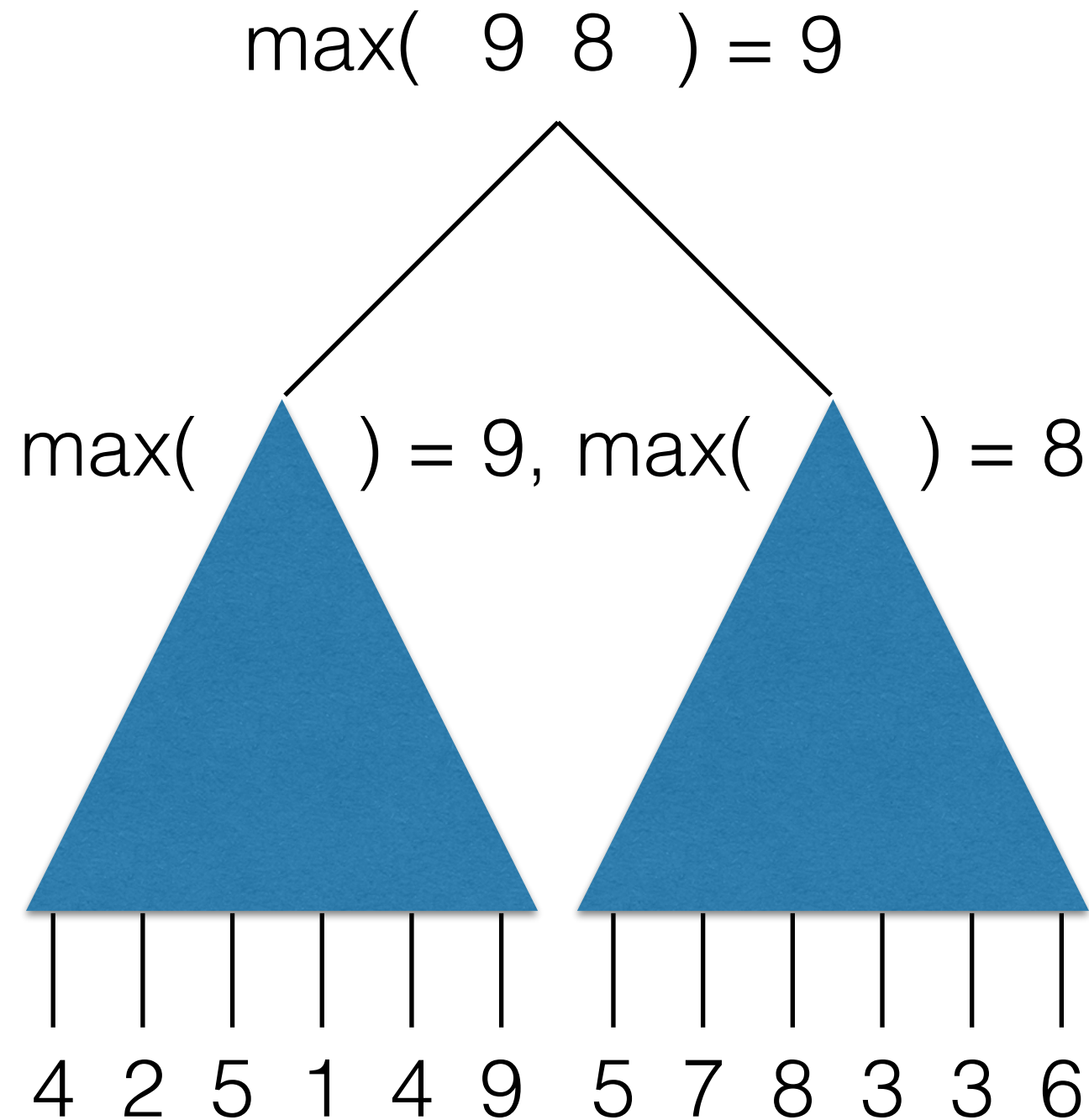
# Using Memo Tables

4 2 5 1 4 9 5 7 8 3 3 6

# Using Memo Tables

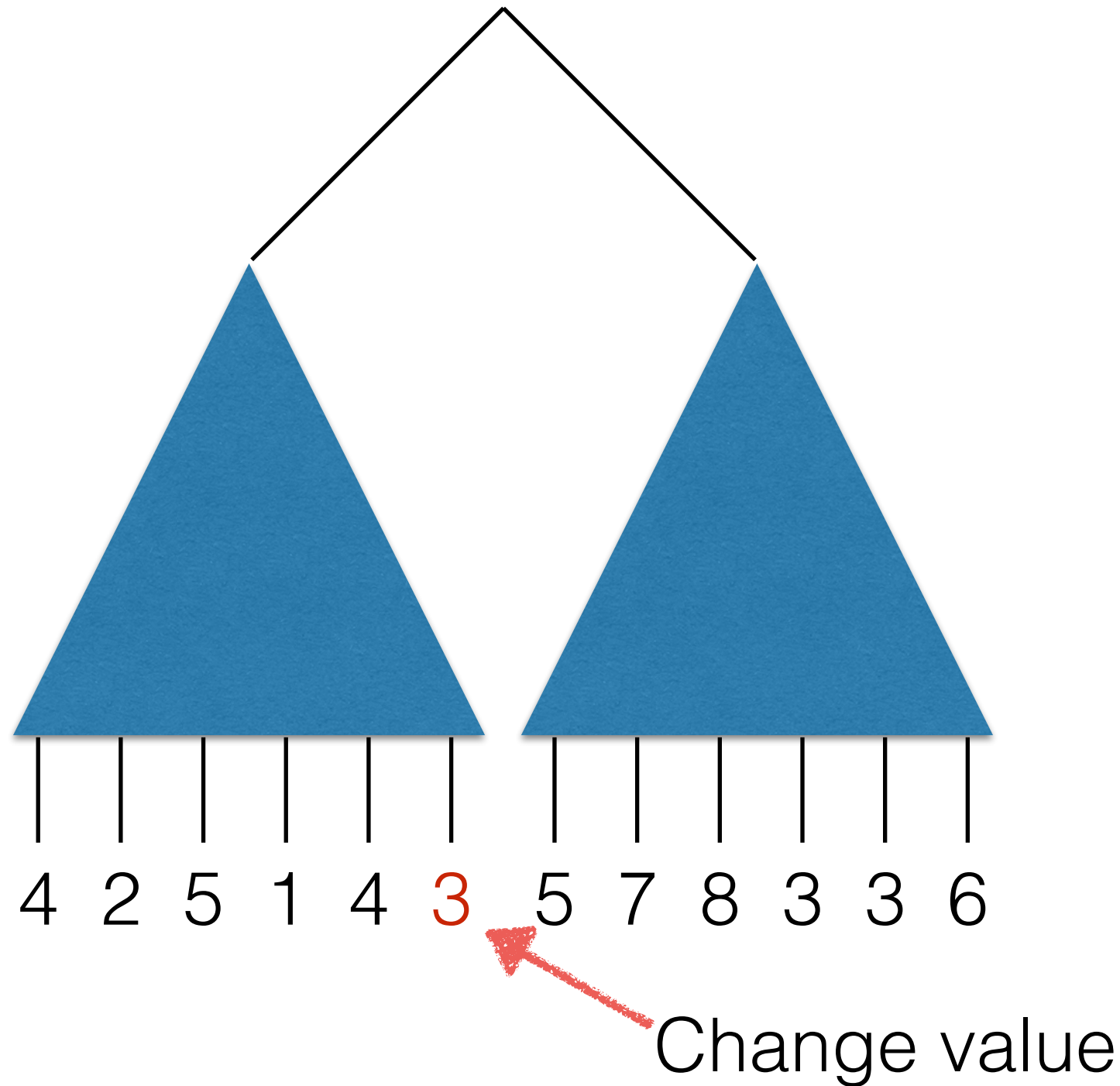


# Using Memo Tables



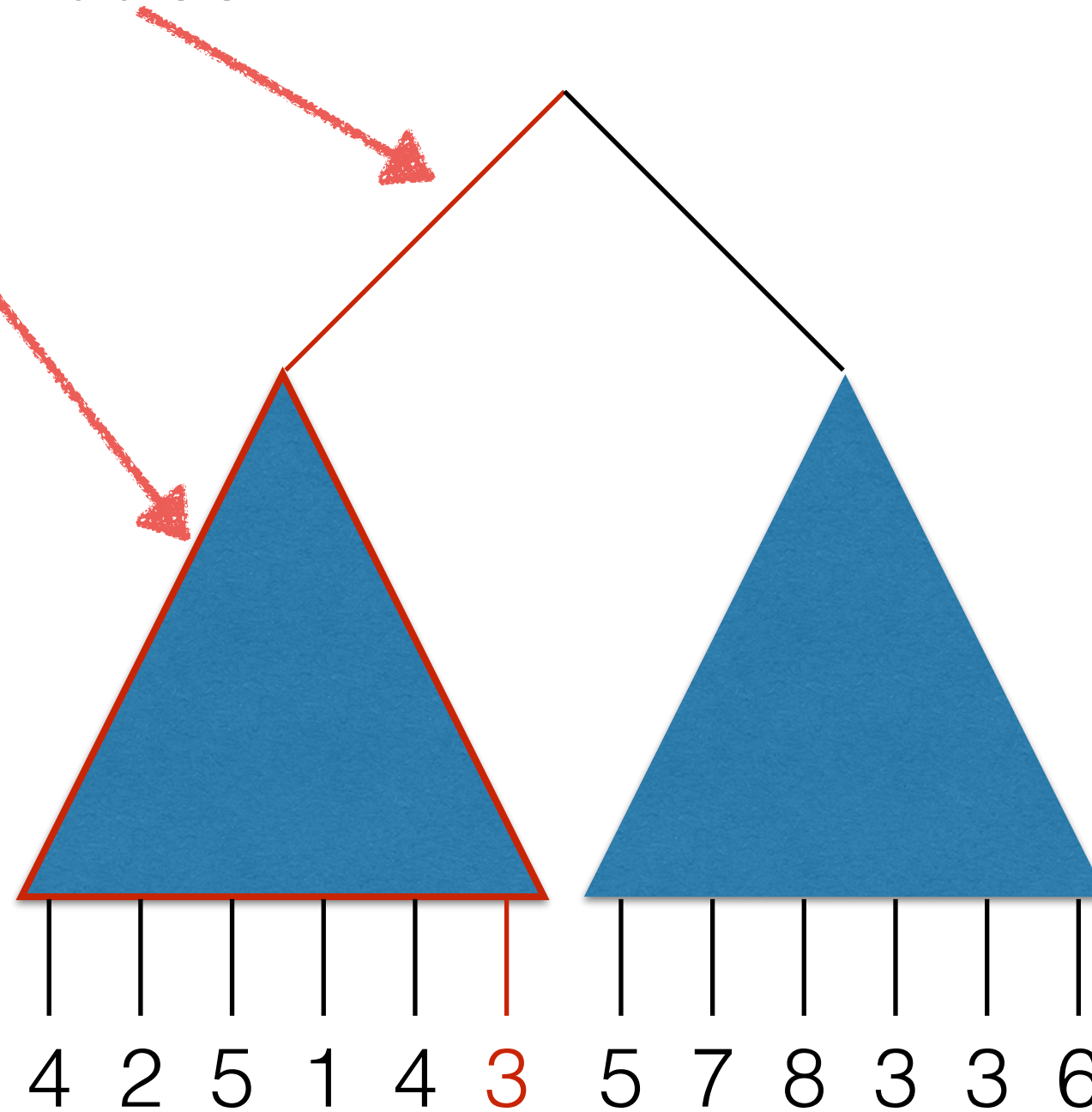
Add results  
to table

# Using Memo Tables



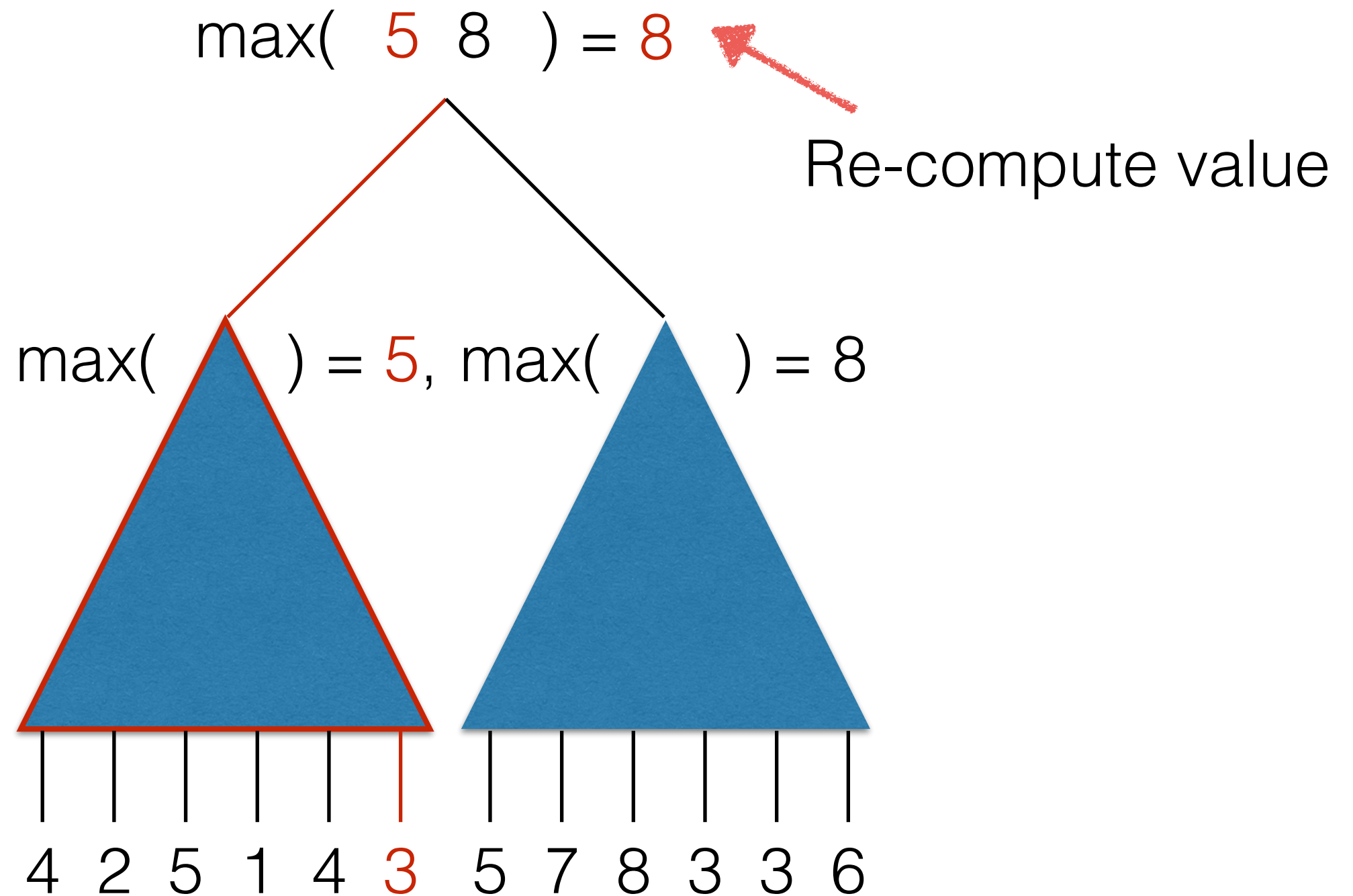
# Using Memo Tables

Update persistent tree





# Using Memo Tables



# Using Memo Tables

$$\max(5, 8) = 8$$

Memo match!

$$\max(\quad) = 5$$

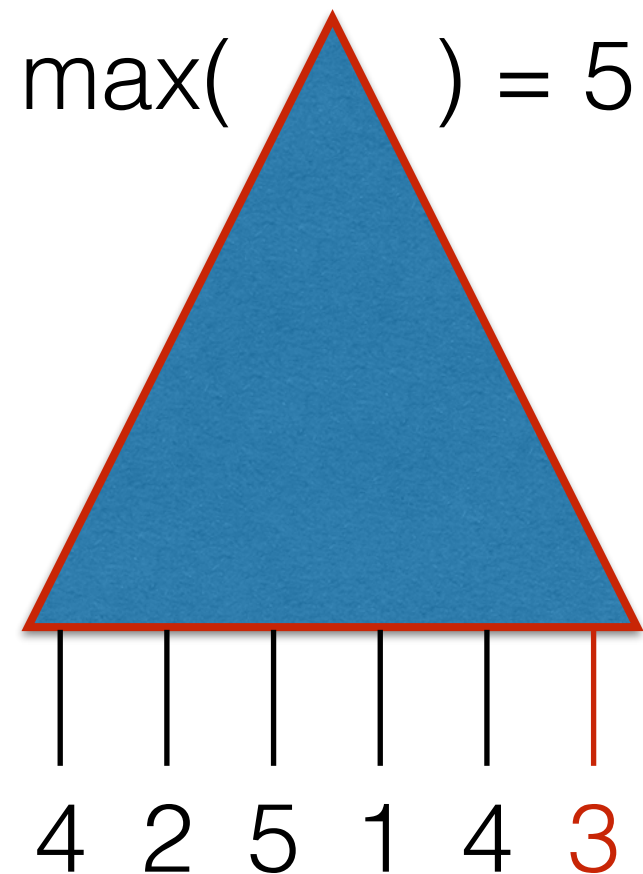
$$\max(\quad) = 8$$

$\max(4, 2, 5, 1, 4, 3)$        $\max(5, 7, 8, 3, 3, 6)$

# Using Memo Tables

But there's a problem with memo tables

# Using Memo Tables



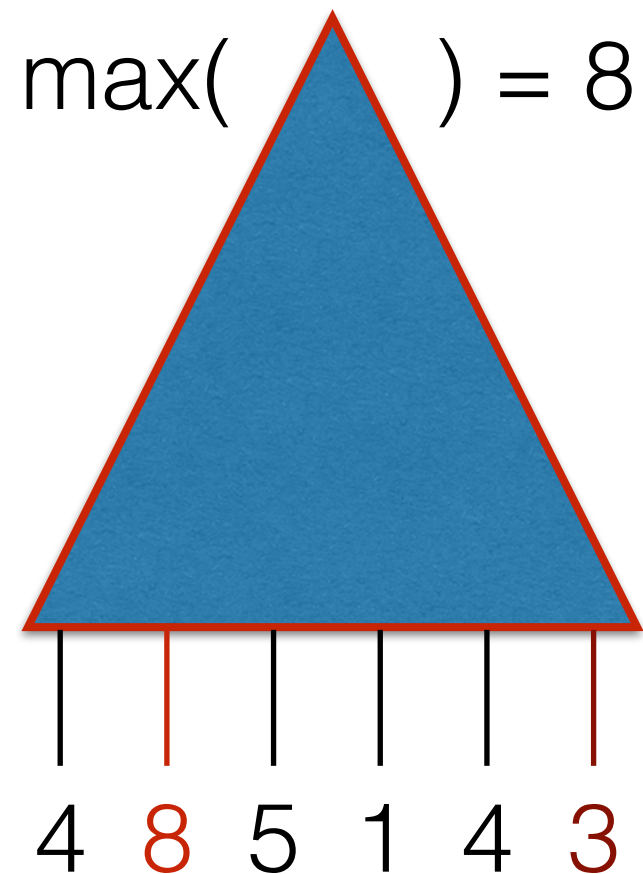
Memo

T1 9

T2 5

Editing a persistent data structure

# Using Memo Tables



Memo

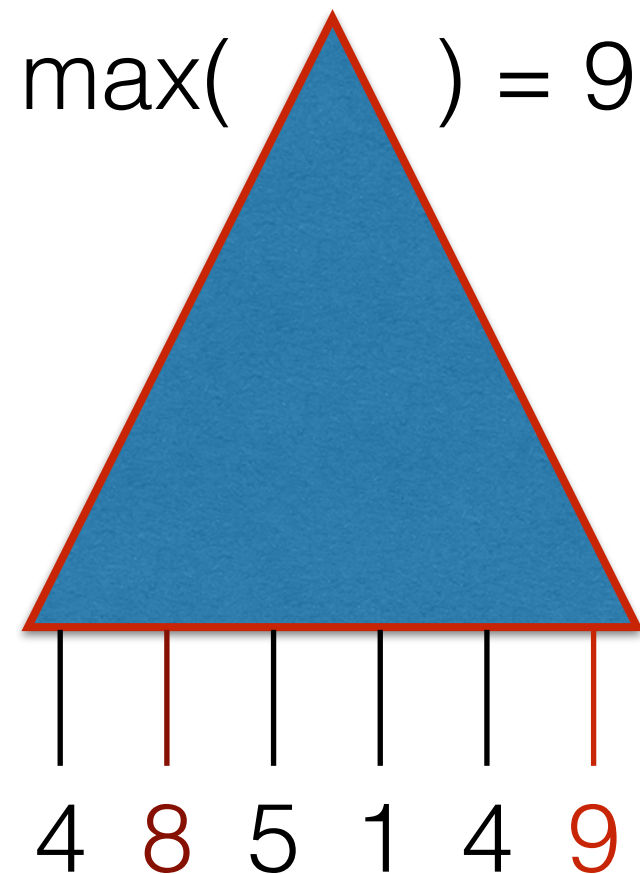
T1 9

T2 5

T3 8

Editing a persistent data structure

# Using Memo Tables



## Memo

T1 9

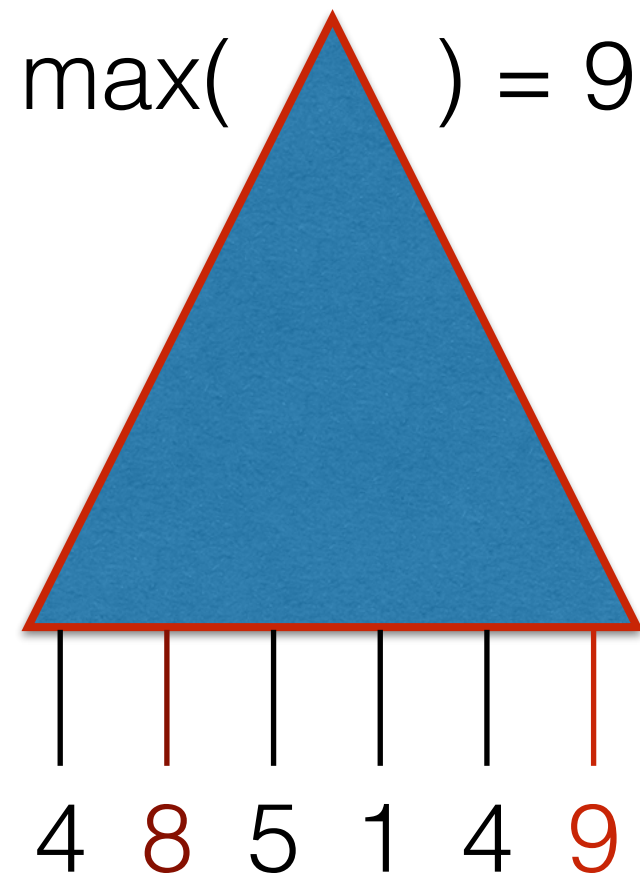
T2 5

T3 8

T4 9

Editing a persistent data structure

# Using Memo Tables



Memo

T1 9

T2 5

T3 8

T4 9

\* \* \*

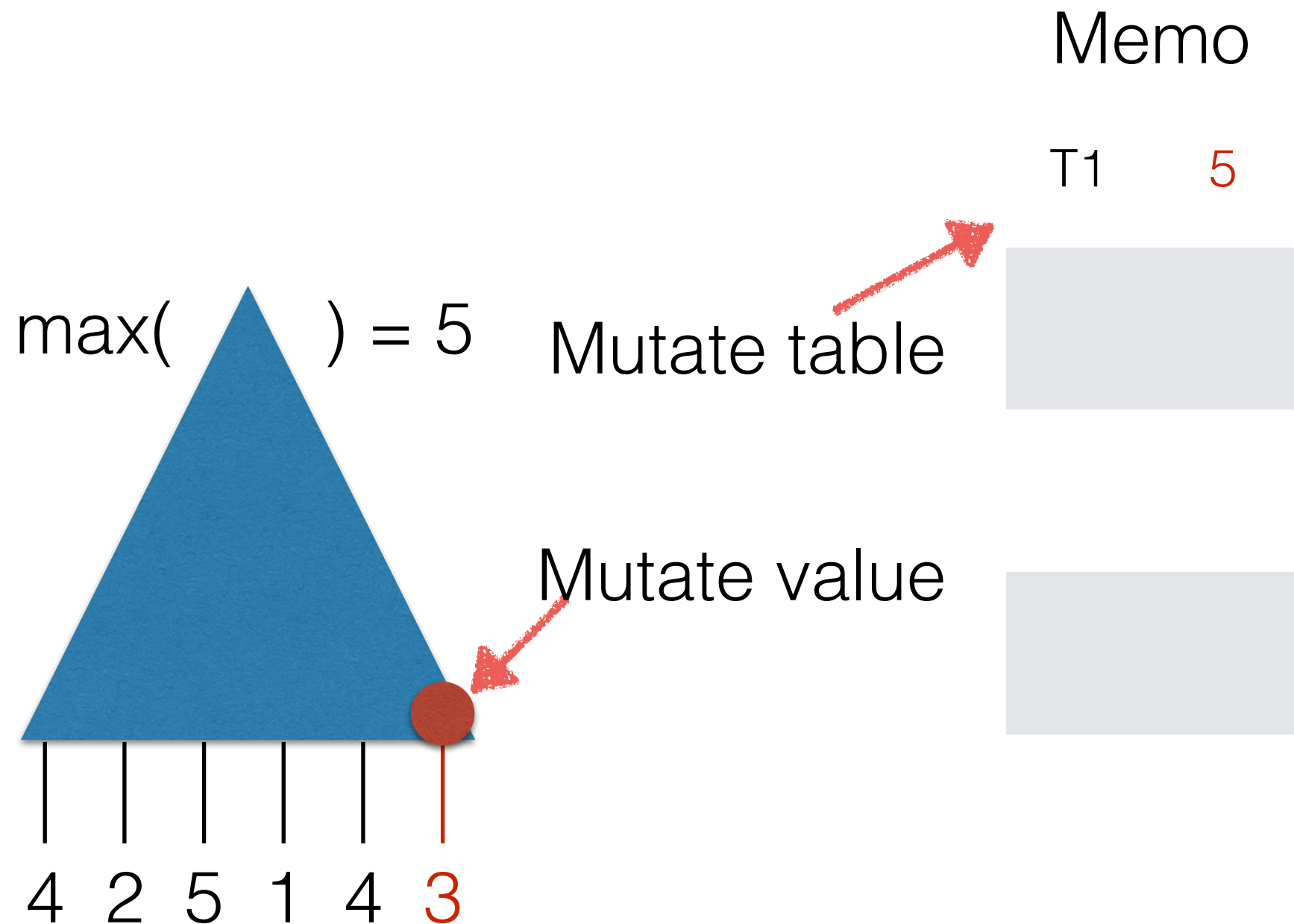
New entry for  
each change

# Using Memo Tables

Lets try mutating values

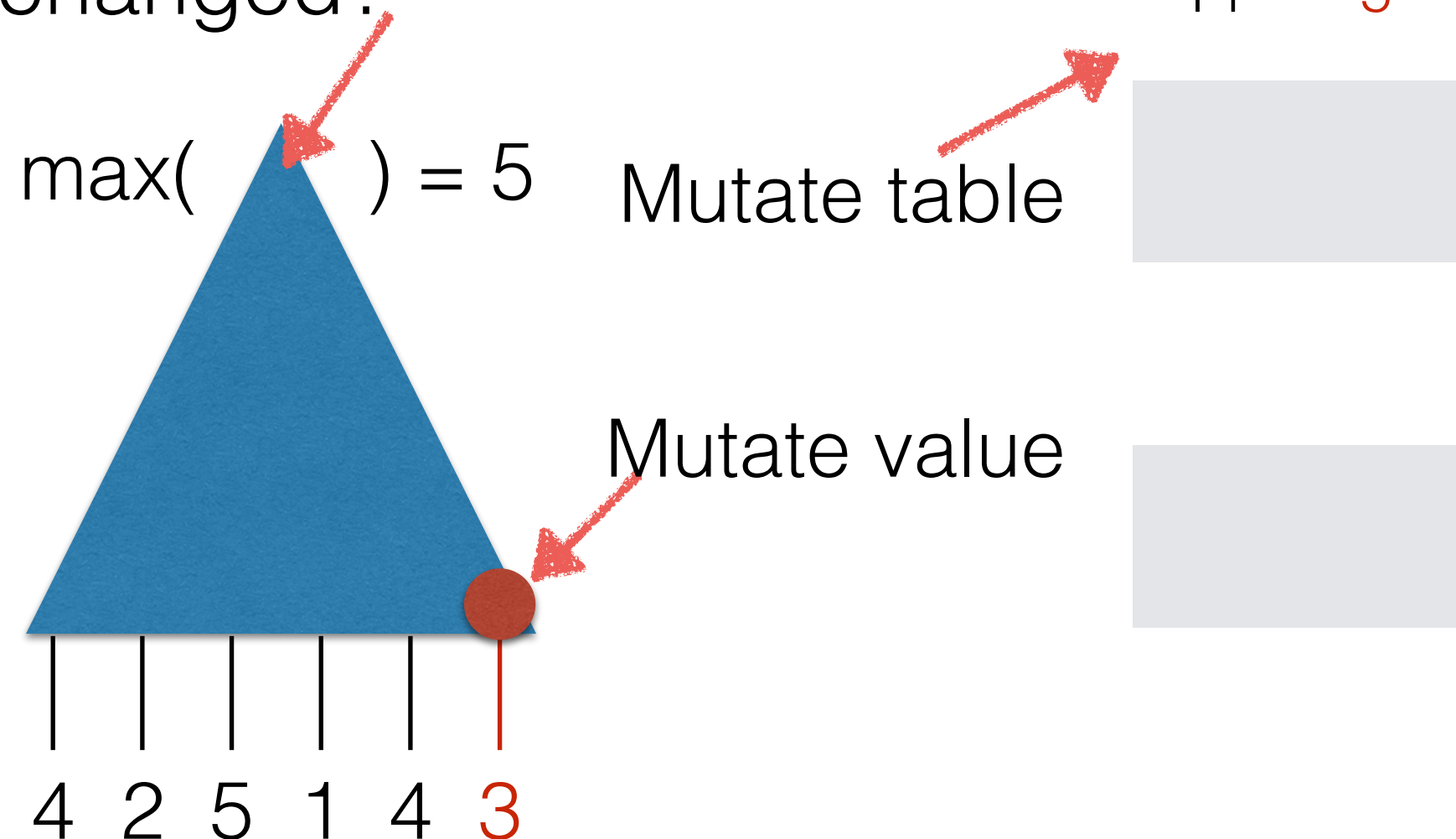


# Using Memo Tables



# Using Memo Tables

But how do we know that the computation result at the root changed?



# Language-based Incremental Computation

Reason about the non-incremental computation

Make calls to library functions for data access

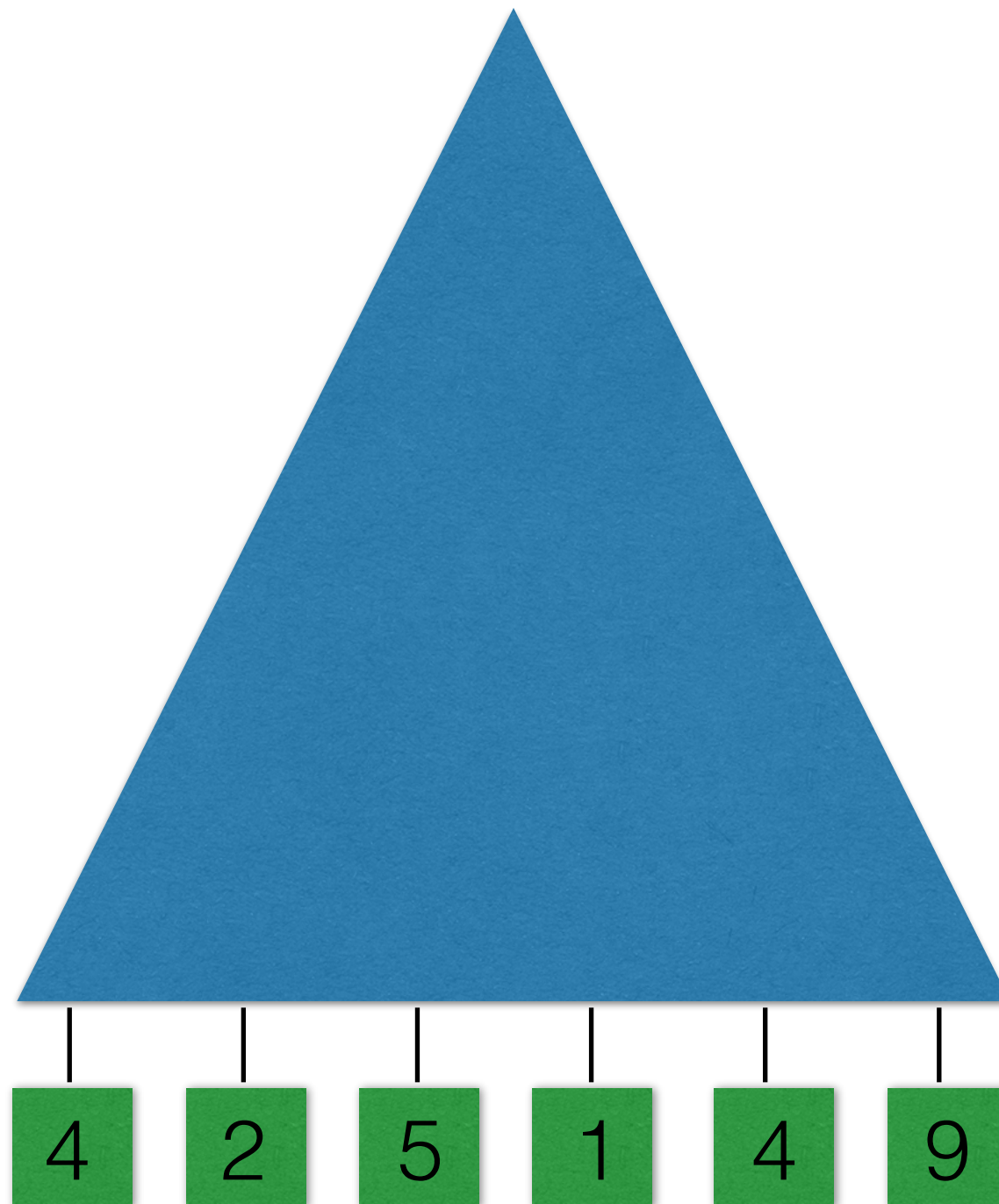
Internally, the library makes use of:

Cached values

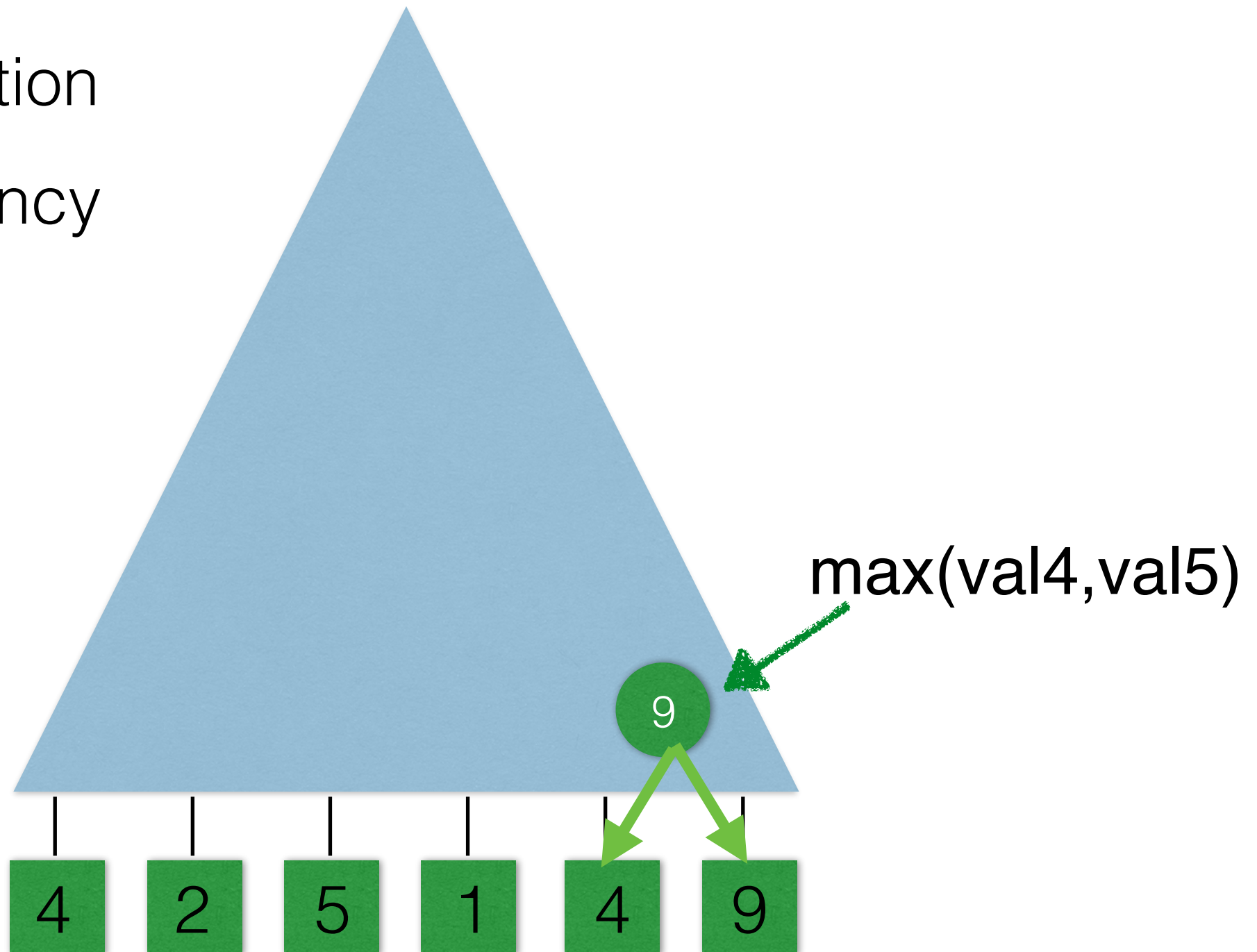
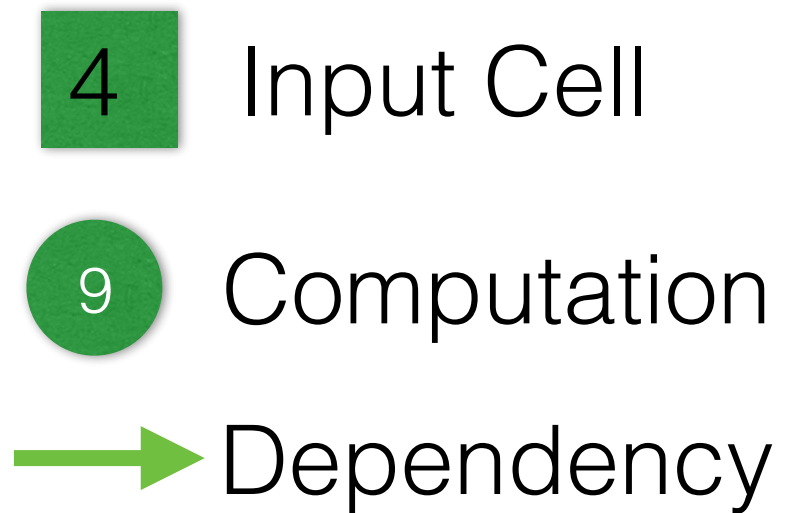
Dependency graphs

# Dependency Graphs

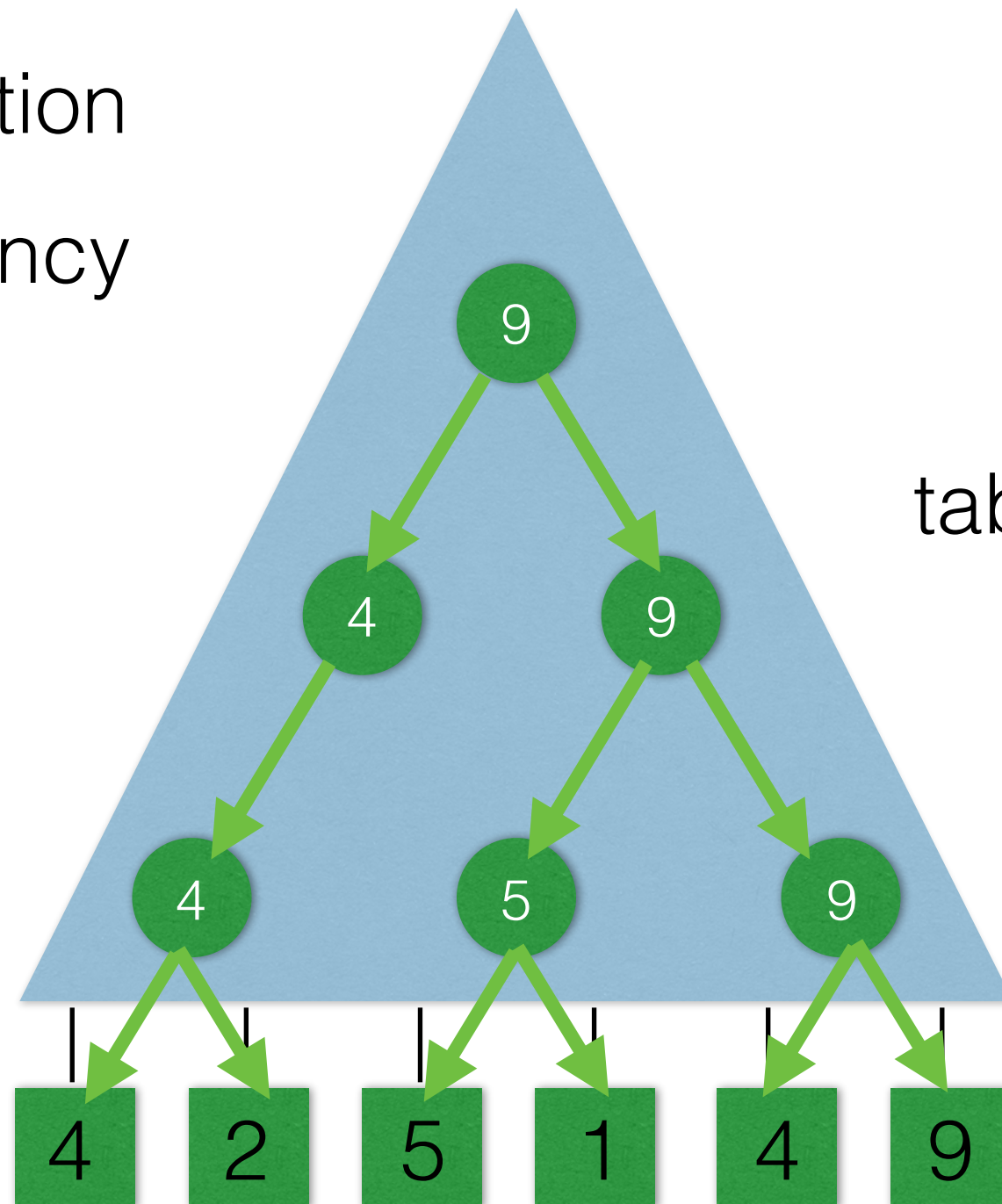
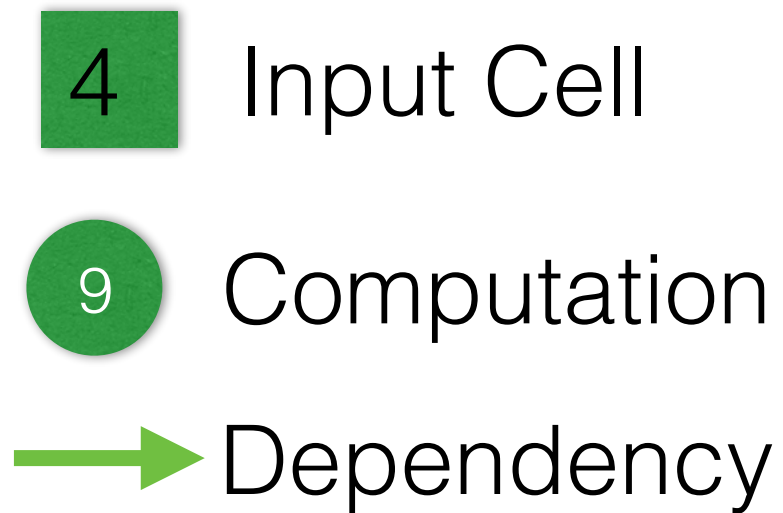
4 Input Cell



# Dependency Graphs

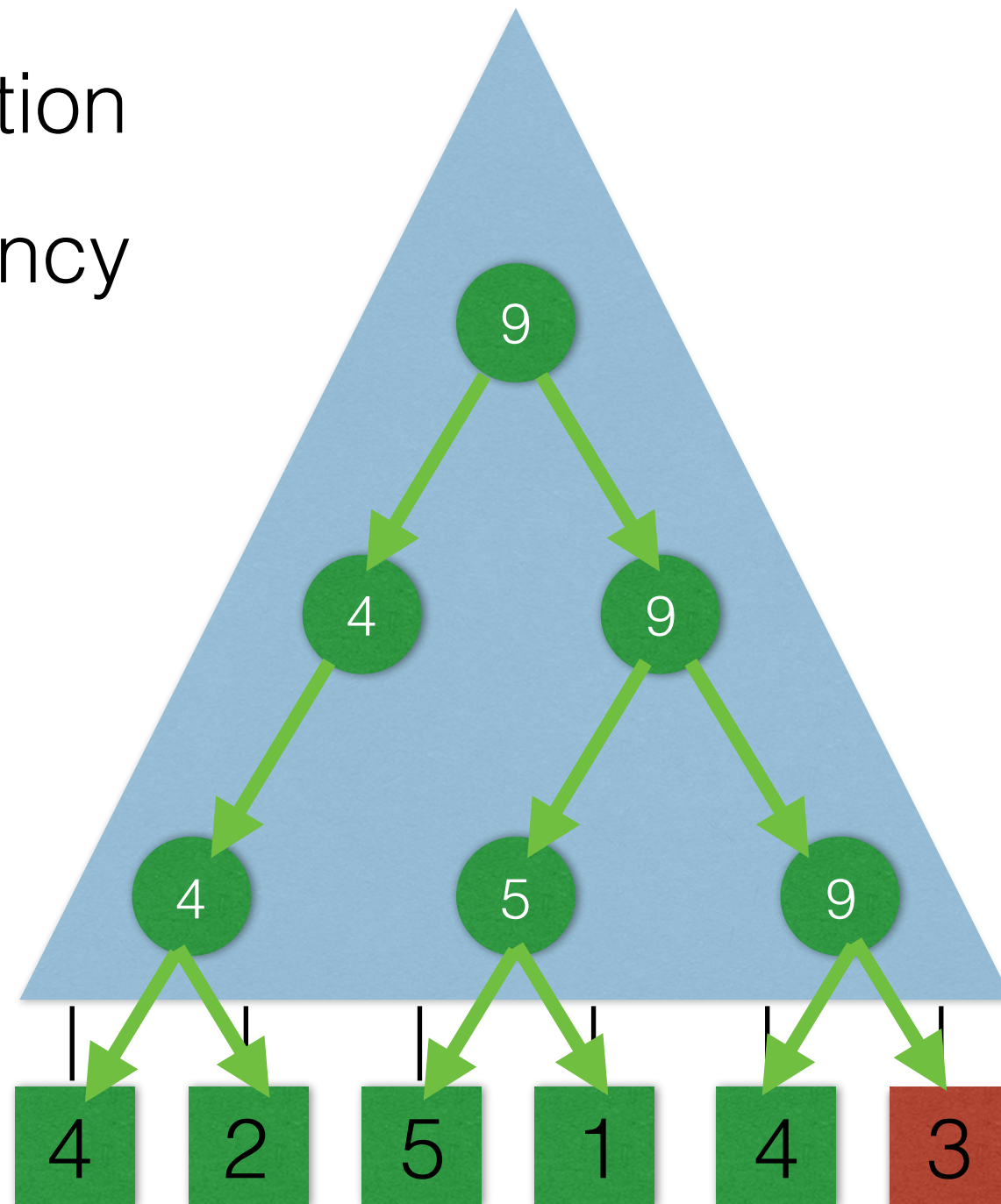
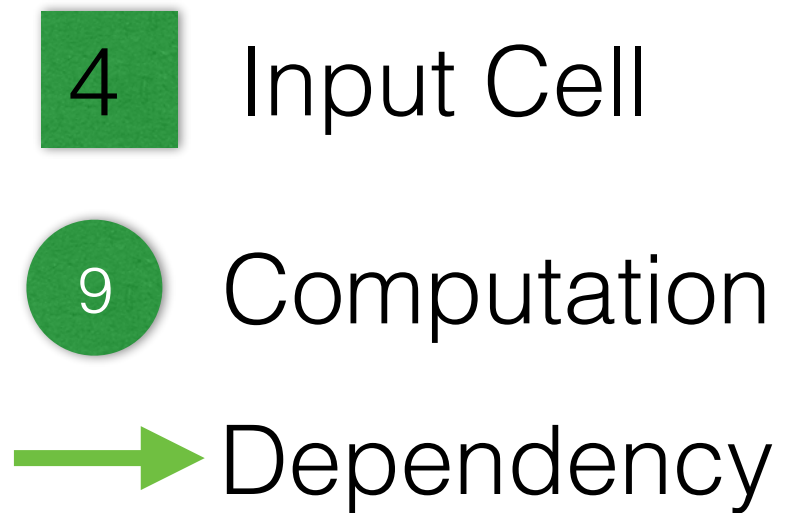


# Dependency Graphs



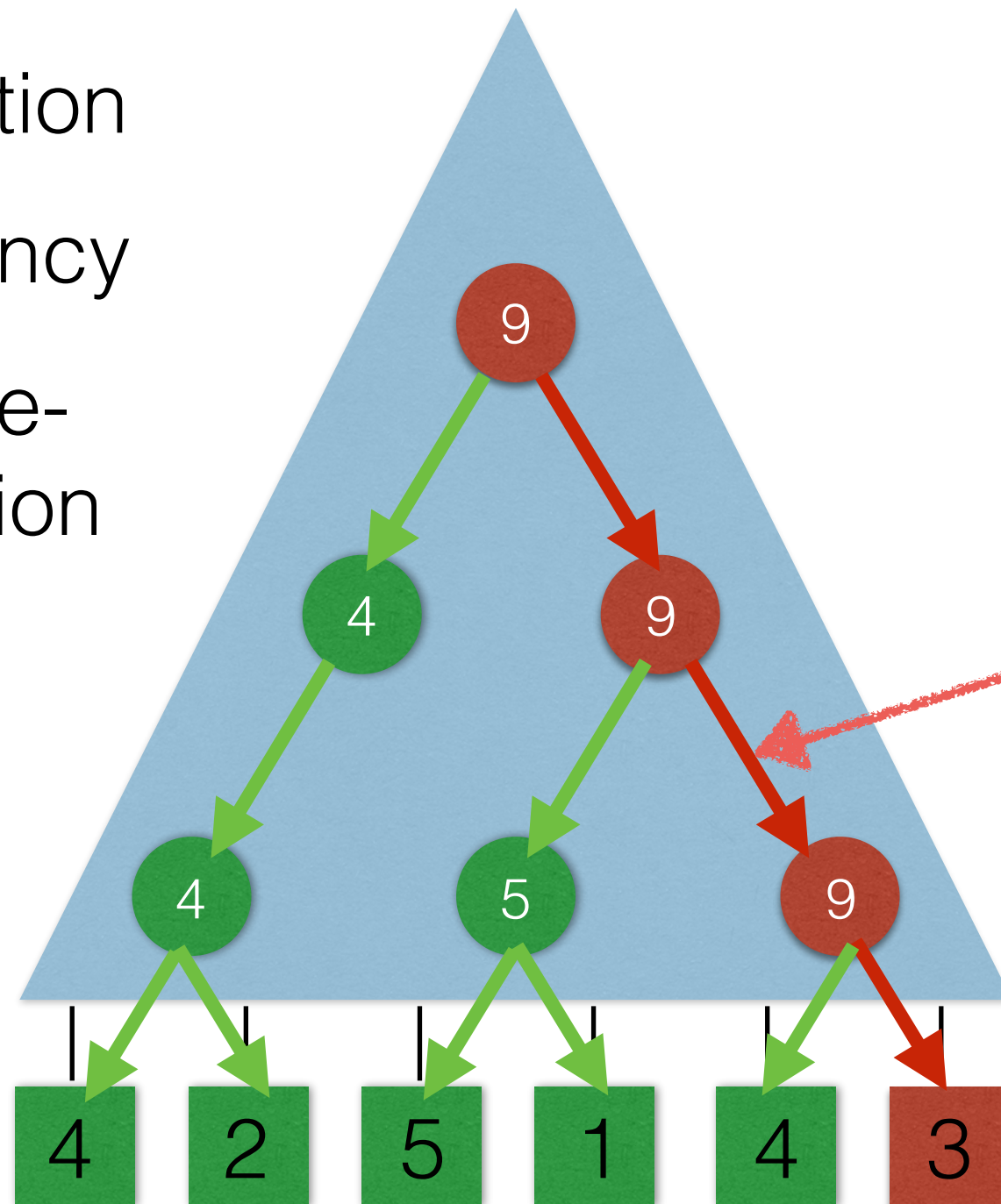
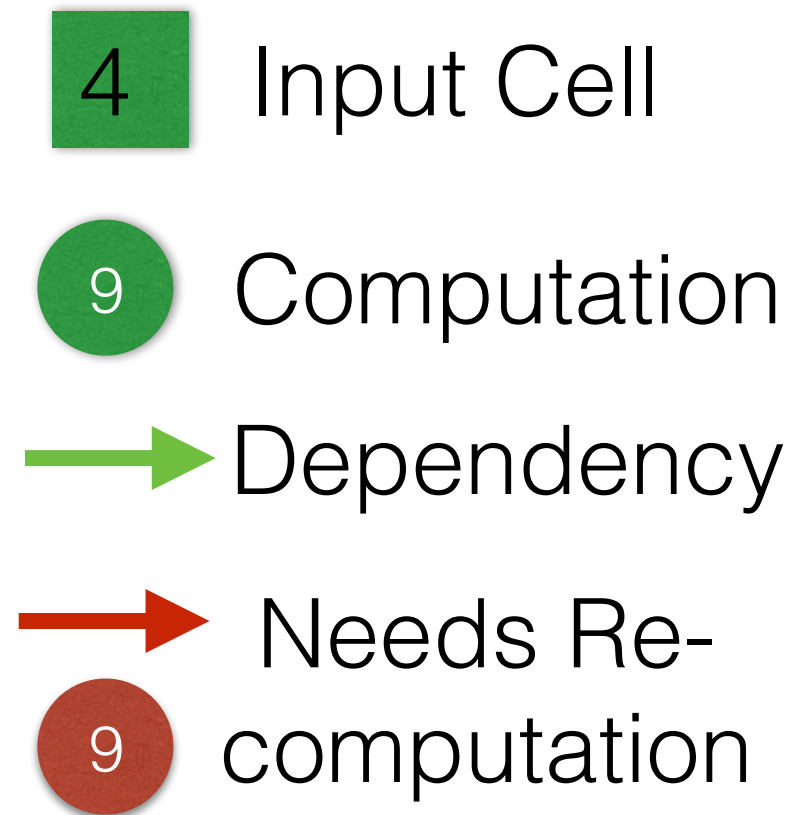
With one memo  
table entry per thunk

# Dependency Graphs



Mutate Value

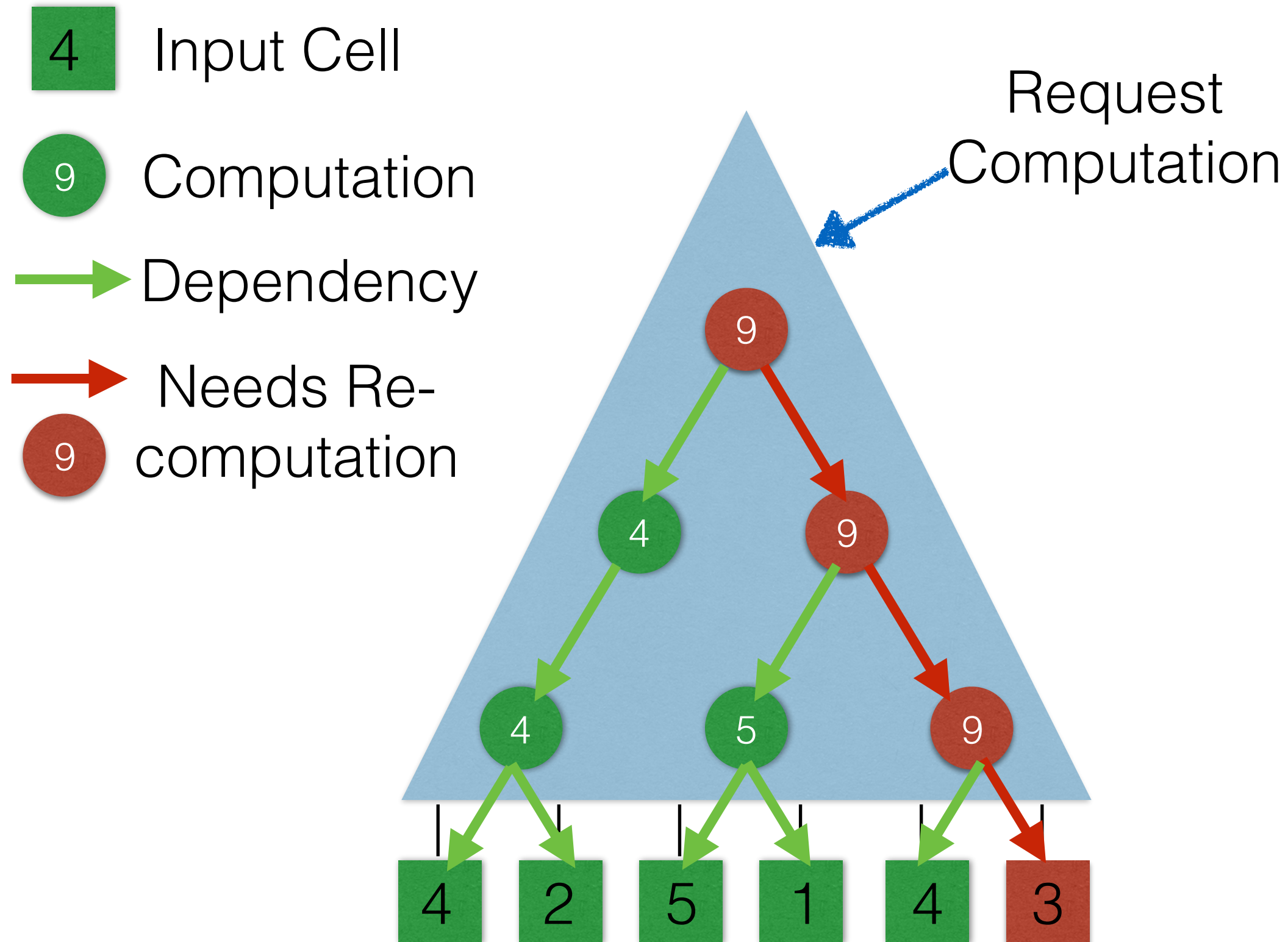
# Dependency Graphs



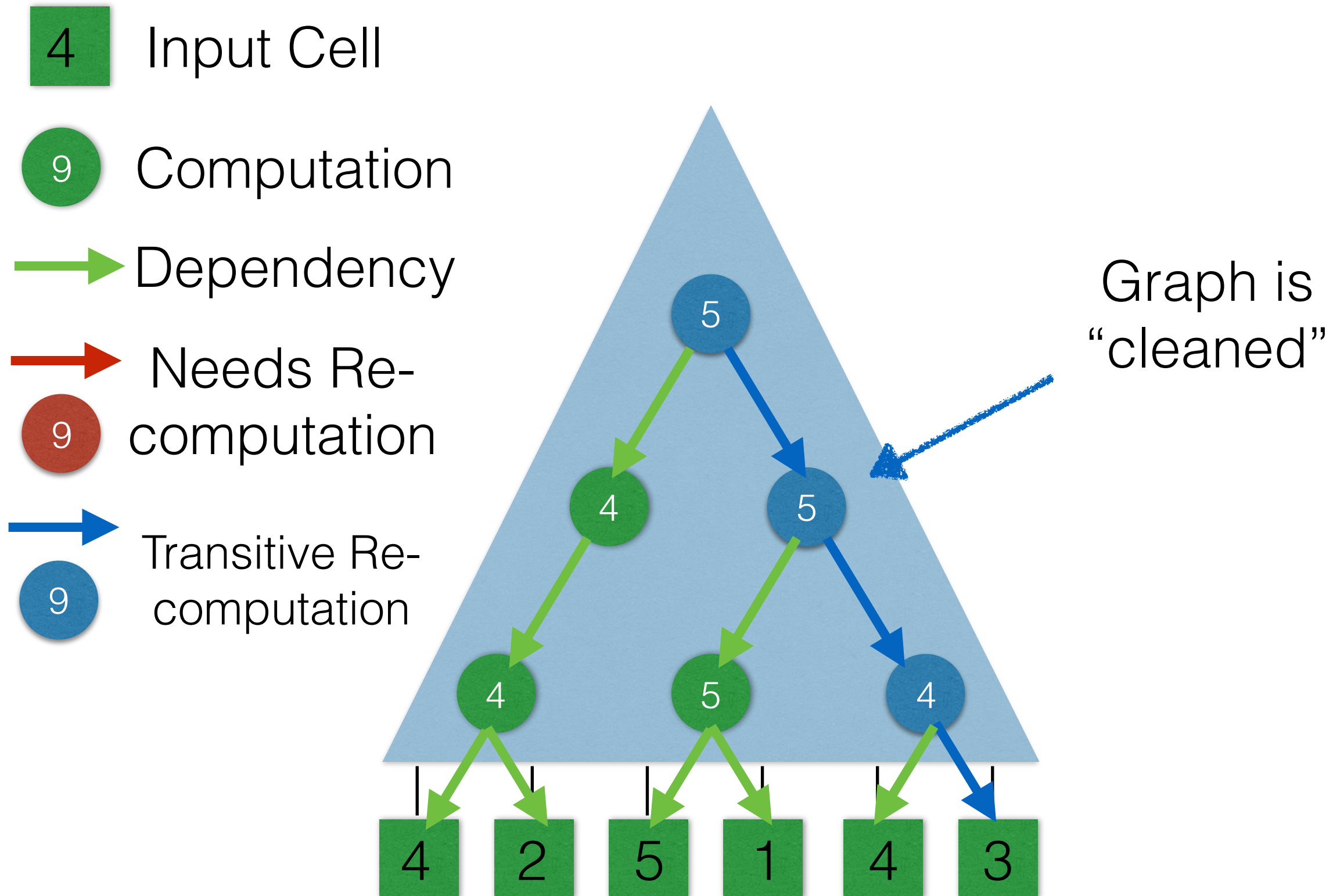
Transitively  
mark graph as  
“dirty”



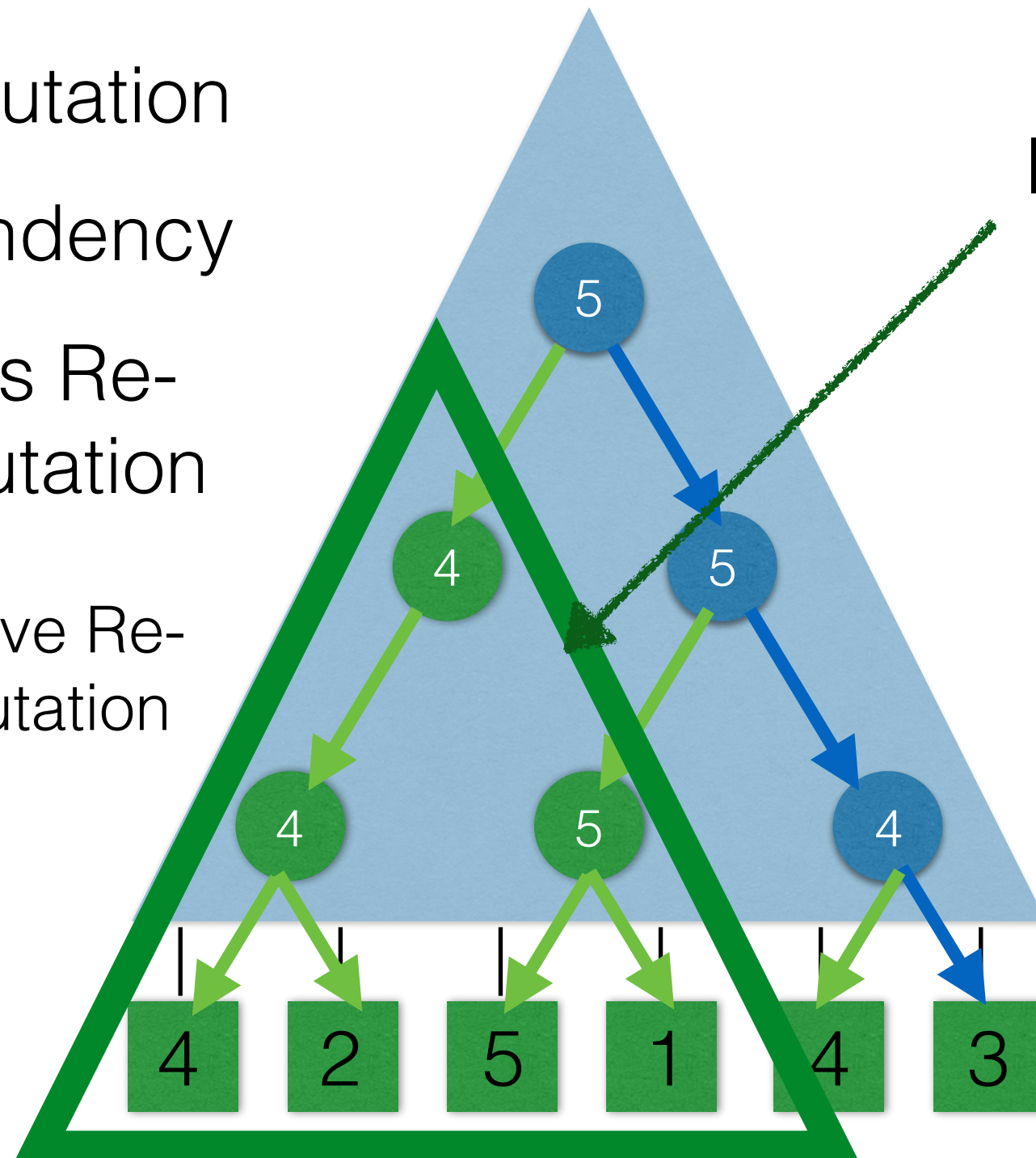
# Dependency Graphs



# Dependency Graphs



# Dependency Graphs



Memo match!

# Research challenge

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

# Speed Problem

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

[www.rust-lang.org](http://www.rust-lang.org)



[Documentation](#)

[Install](#)

[Community](#)

[Cont](#)

**Rust** is a systems programming language that runs blazingly fast, prevents segfaults, and guarantees thread safety.

[Inst](#)

# Speed Problem

Incremental performance improved

Non-Incremental performance improved a lot more

# Speed Problem

Computers are better at adding and subtracting numbers than walking through memory

# Speed Problem

Computers are better at adding and subtracting numbers than walking through memory

Incremental computation libraries manage a lot of memory



# Simplicity Problem

How do we advance the use of incremental computation, further simplifying code creation and providing speedups over realistic code?

# Simplicity Problem

Memo tables reduce computations

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Memo tables reduce computations

Dependency graphs hide the memo tables

# Simplicity Problem

Memo tables reduce computations

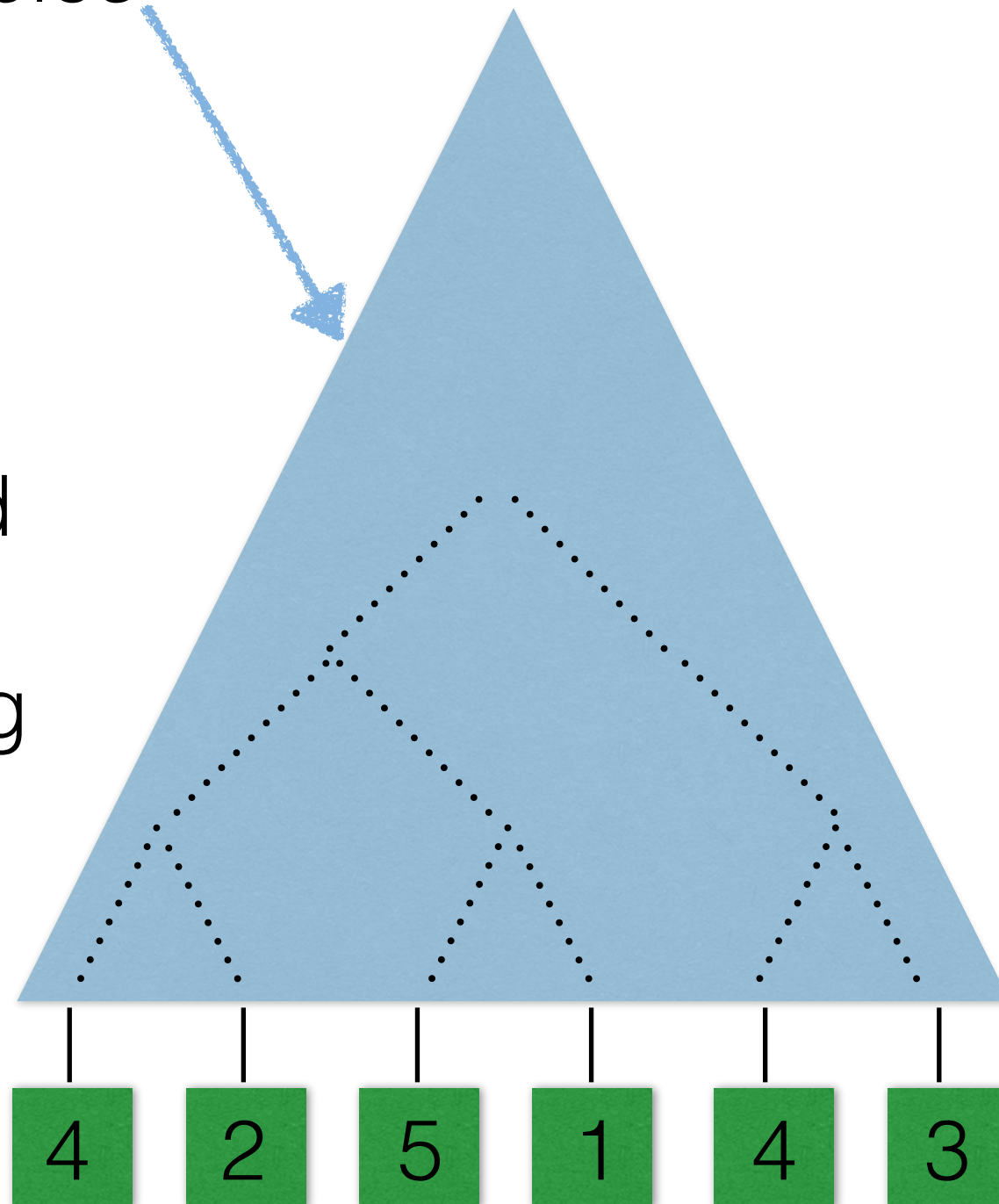
Dependency graphs hide the memo tables

We want a way to generate graphs of similar computations

# Simplicity Problem

Ad-Hoc tree  
and tables

User writes  
trees and  
memo-table  
functions and  
edits by  
reconstructing  
the tree



T1	9
T2	5
.....	.....

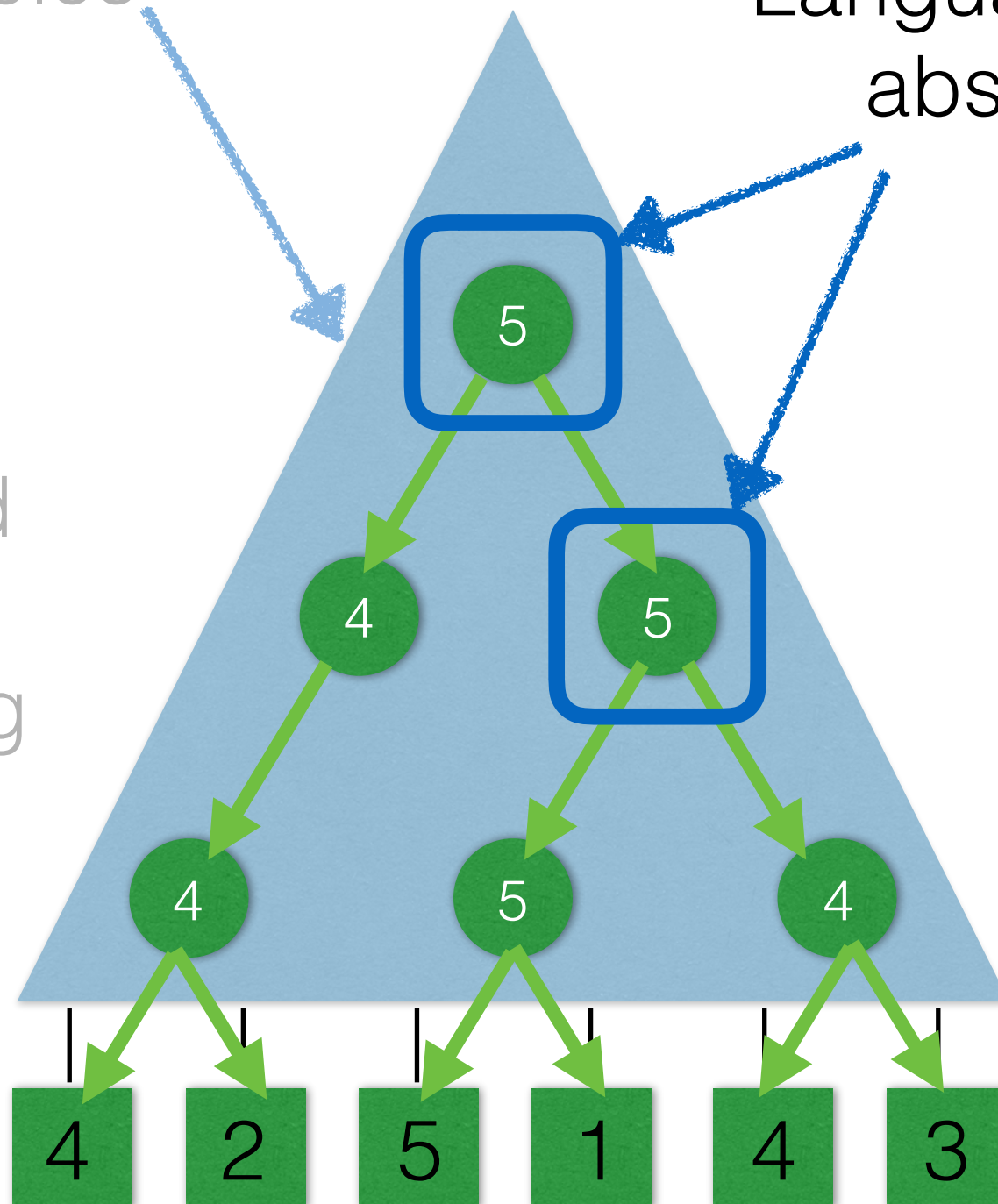
# Simplicity Problem

Ad-Hoc tree  
and tables

Language-based  
abstraction

User writes  
trees and  
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edits by  
reconstructing  
the tree

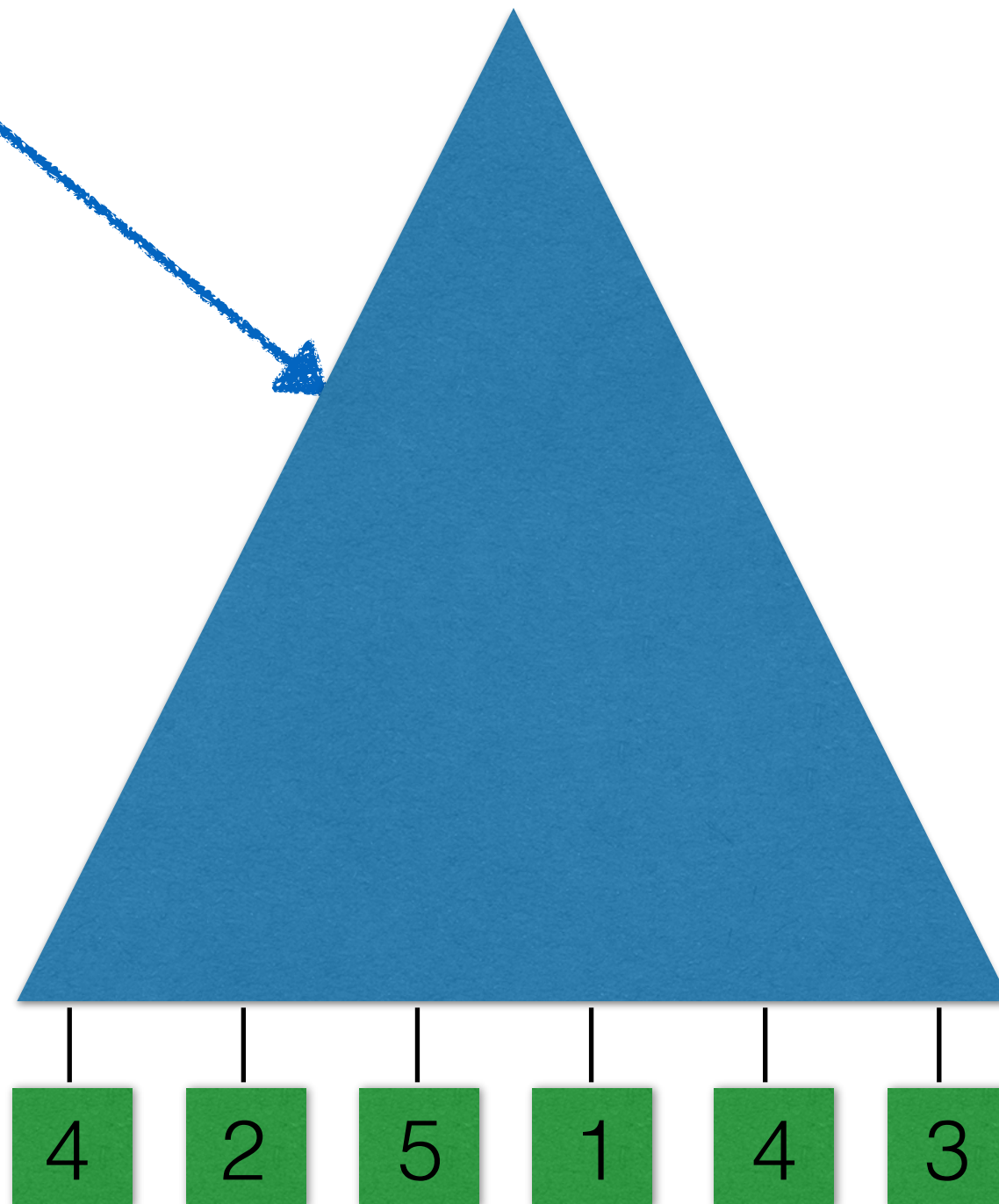
User writes  
functions using  
an inc lib, and  
edits by  
changing single  
values



# Simplicity Problem

Data collections  
abstraction

`seq.fold_up(max) = 5`



User writes non-  
incremental  
functions for  
higher-order  
combinators and  
edits through  
intuitive  
collections  
interface

# Iodyn

Incremental Collections Library  
Based on Adapton

[github.com/cuplv/iodyn.rust](https://github.com/cuplv/iodyn.rust)

rust crate: iodyn

IRaz - Incremental sequences  
(Giraz?, Sigraz?)

In progress: Tries, Graphs



# Giraz

Incremental Sequence data structure  
Based on Adapton

Based on RAZ data structure

Includes incremental functions

Insert

Delete

Move cursor

fold\_up -

tree fold, compute at leaf and  
binary nodes

fold\_lr -

list fold, compute at each element

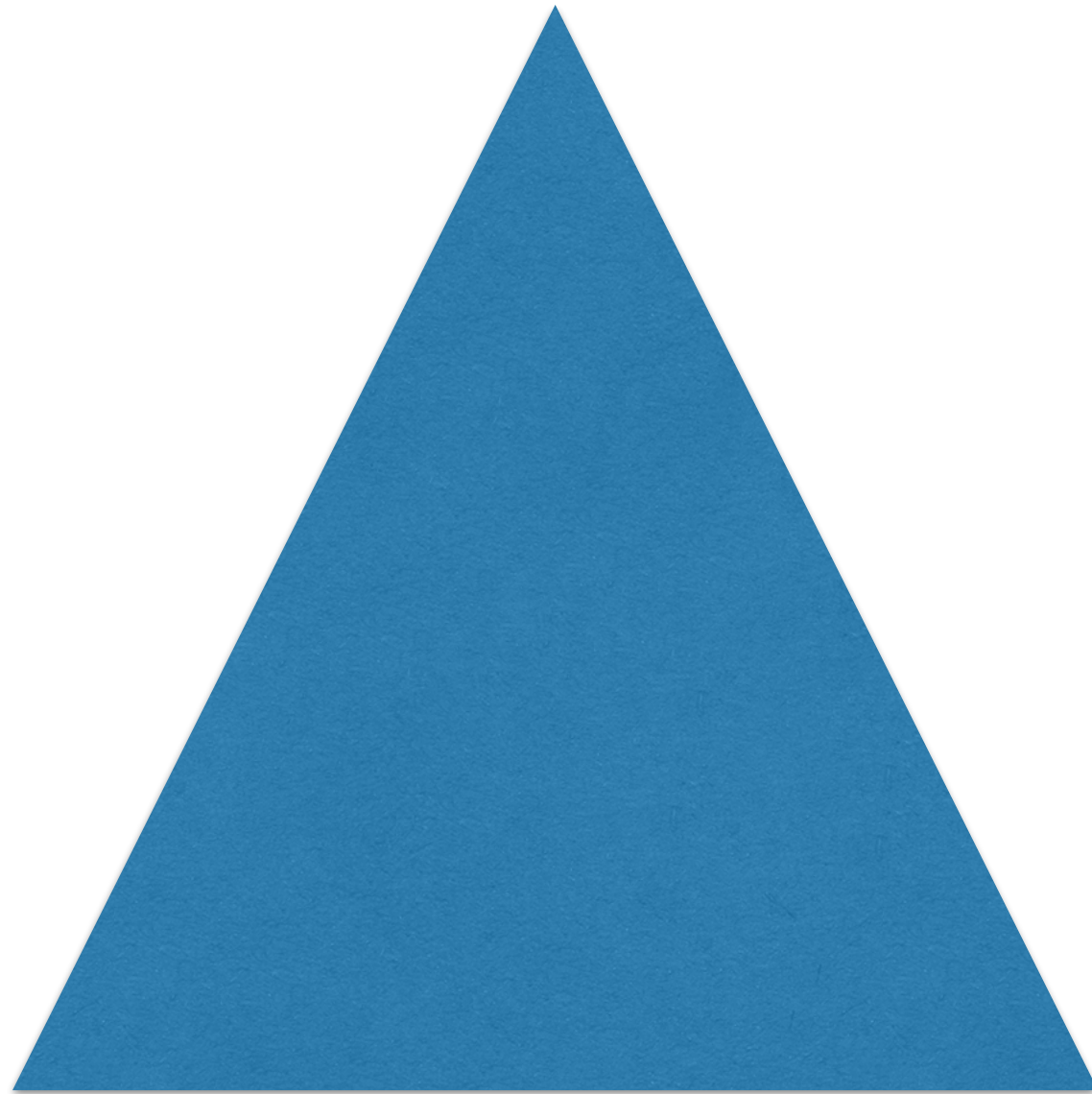
map -

maintain structure and transform each  
element

# What is a Giraz?

Sequence: 

# What is a Giraz?



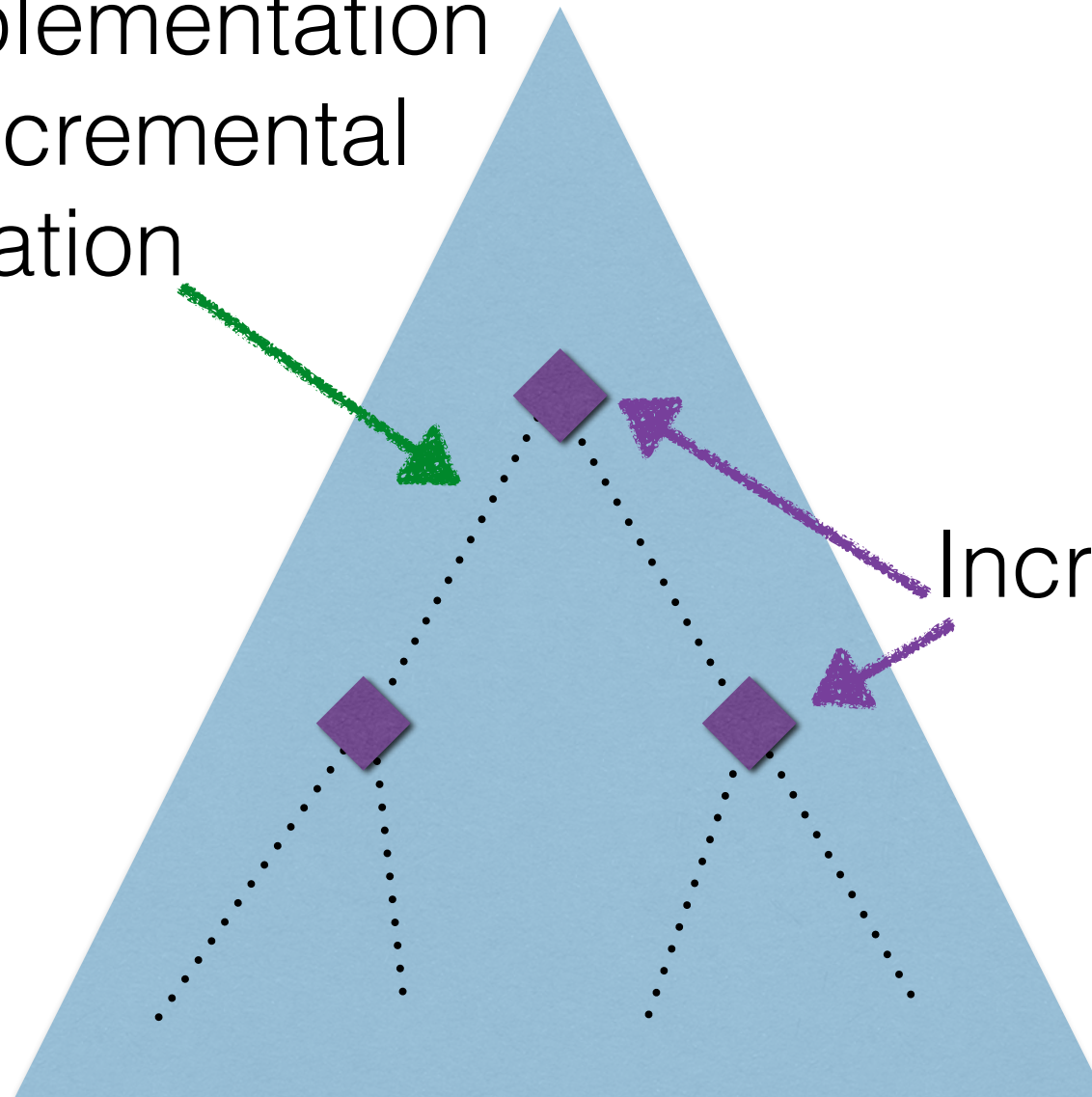
Sequence:





# What is a Giraz?

# Tree-based implementation for efficient incremental computation



# Incremental meta-data at tree nodes

Sequence:



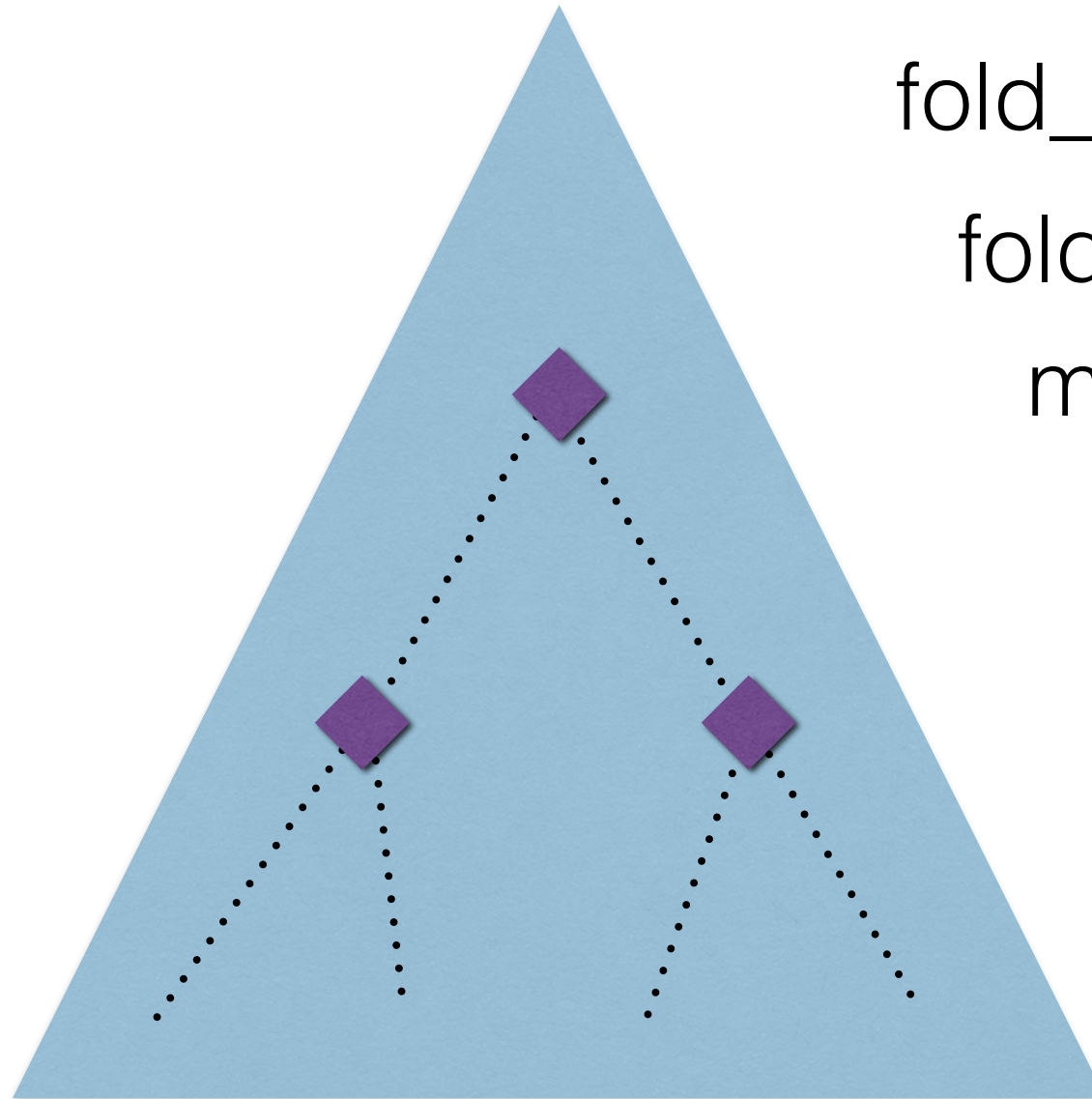
# What is a Giraz?

Higher-order collections combinators

fold\_up()

fold\_lr()

map()

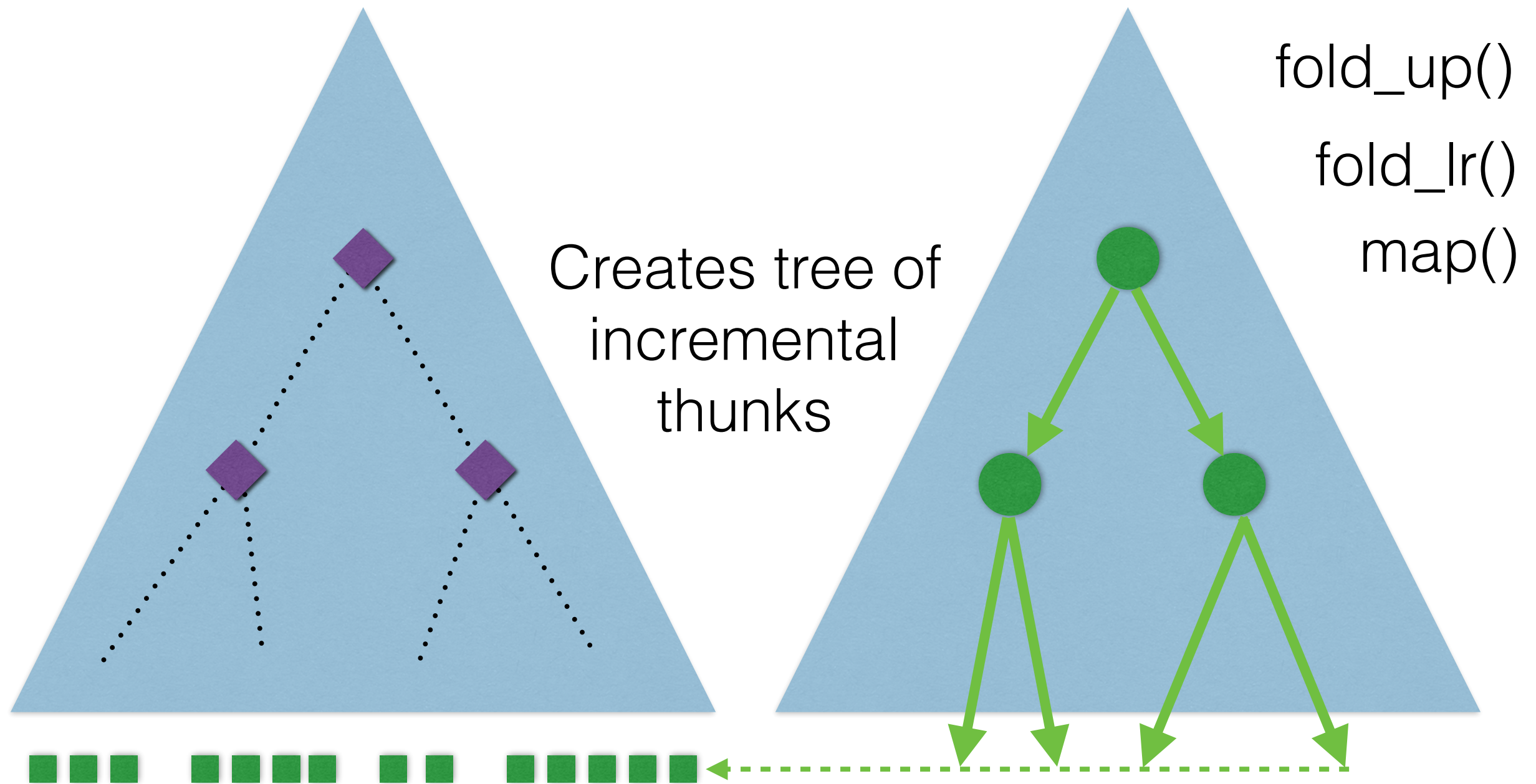


Sequence:



# What is a Giraz?

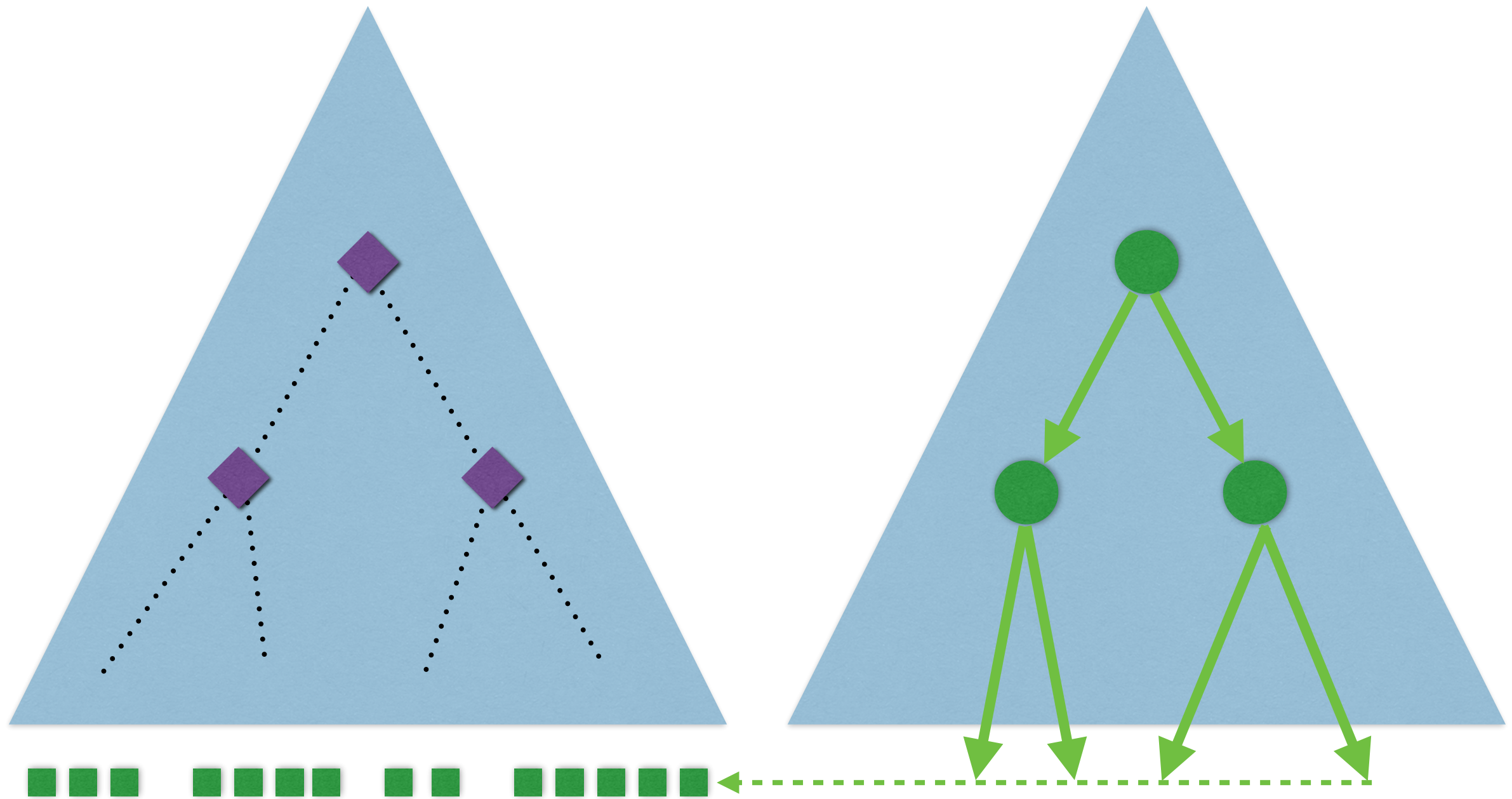
## Higher-order collections combinators





# What is a Giraz?

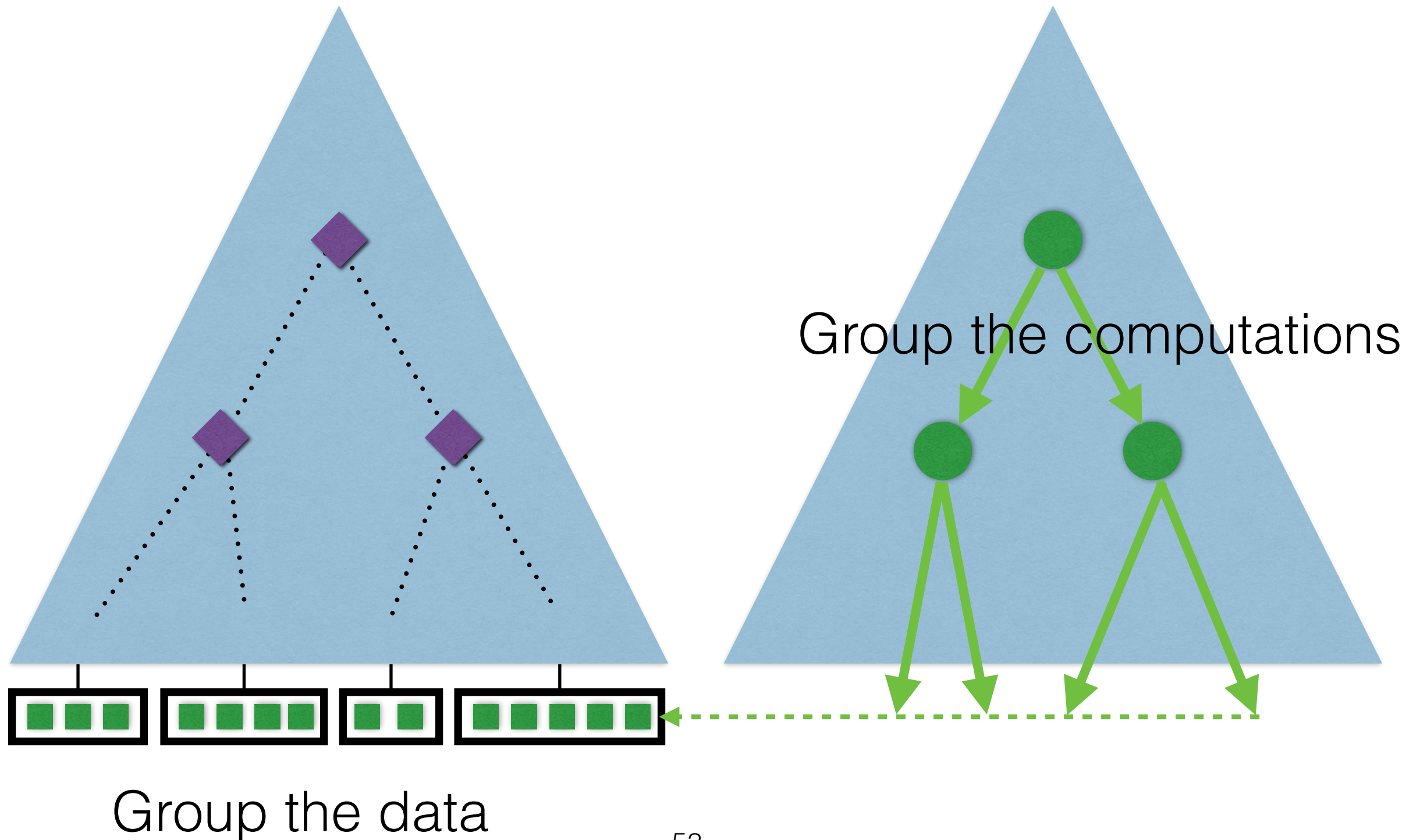
Allows user control over subproblem size





# What is a Giraz?

Allows user control over subproblem size



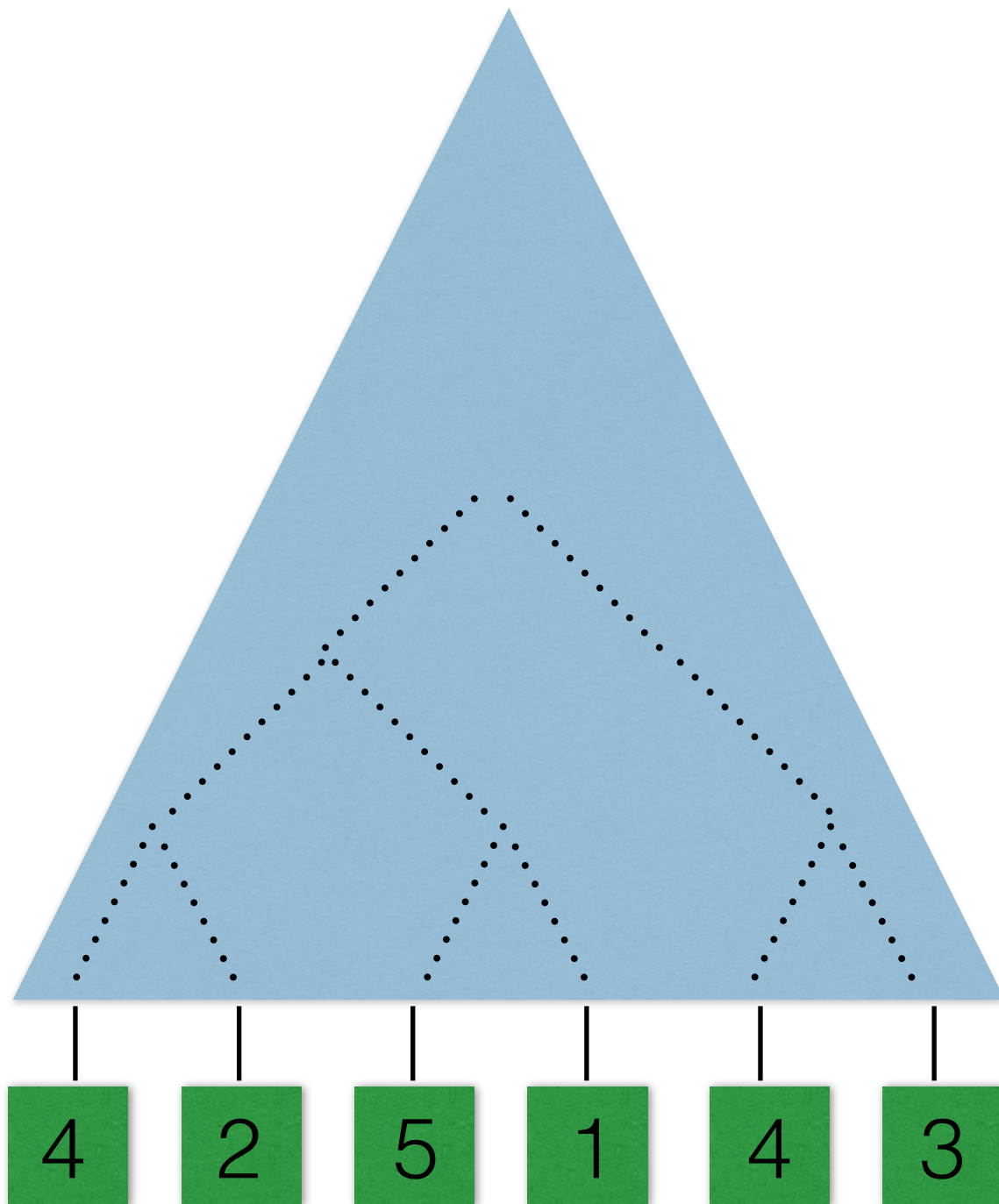
# Overhead of incremental computation

Computers are better at adding and subtracting numbers than walking through memory

With optimized code, it's faster to re-compute subproblems than to manage them through our dependency graphs

# Overhead of incremental computation

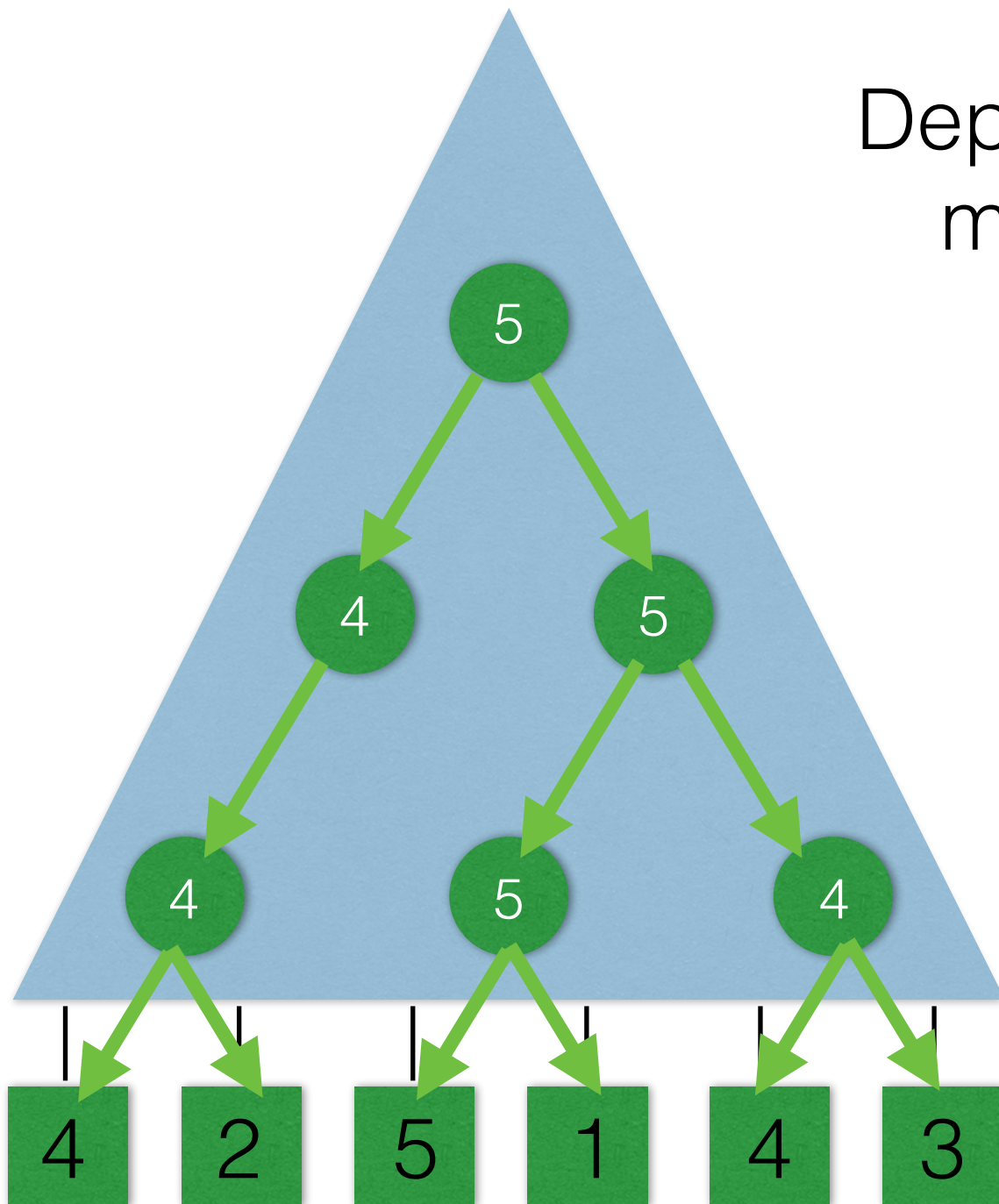
Ad-hoc: many memo-entries per change



# Overhead of incremental computation

Ad-hoc: many memo-entries per change

Dependency graphs:  
memo-entry per computation

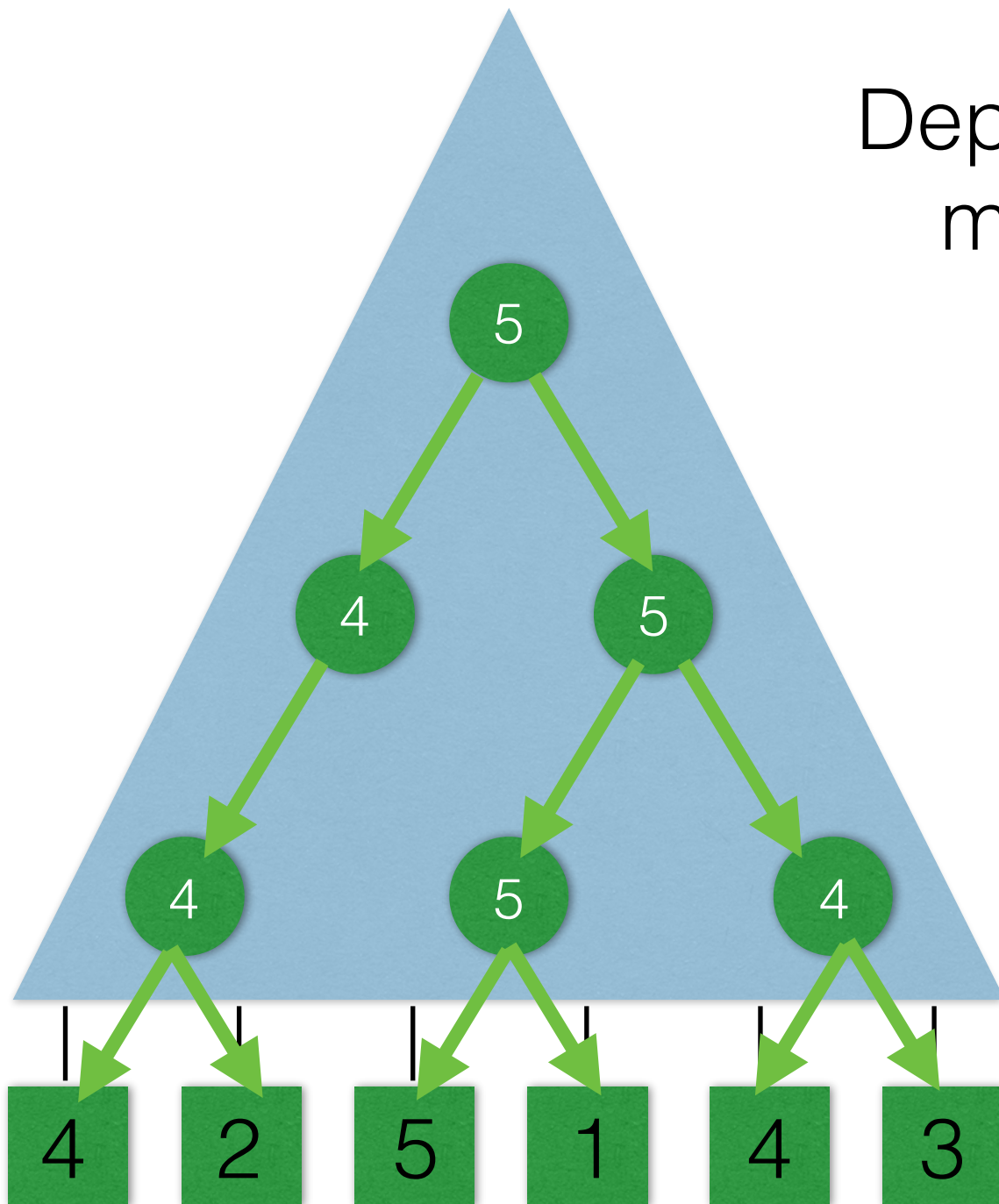


# Overhead of incremental computation

Ad-hoc: many memo-entries per change

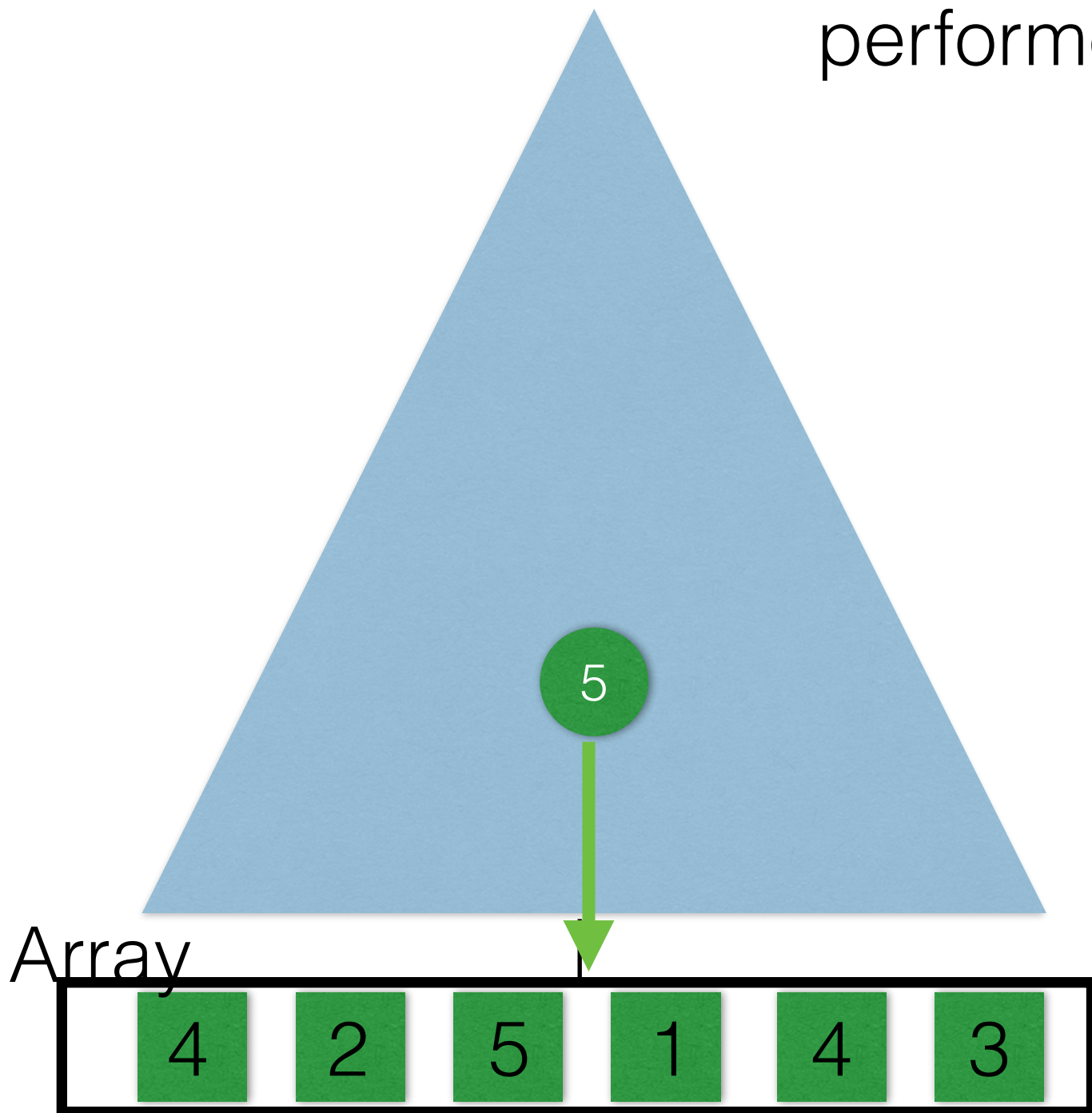
Dependency graphs:  
memo-entry per computation

Need to reduce further



# Reducing the overhead of incremental computation

One memorized computation is performed over an array of data



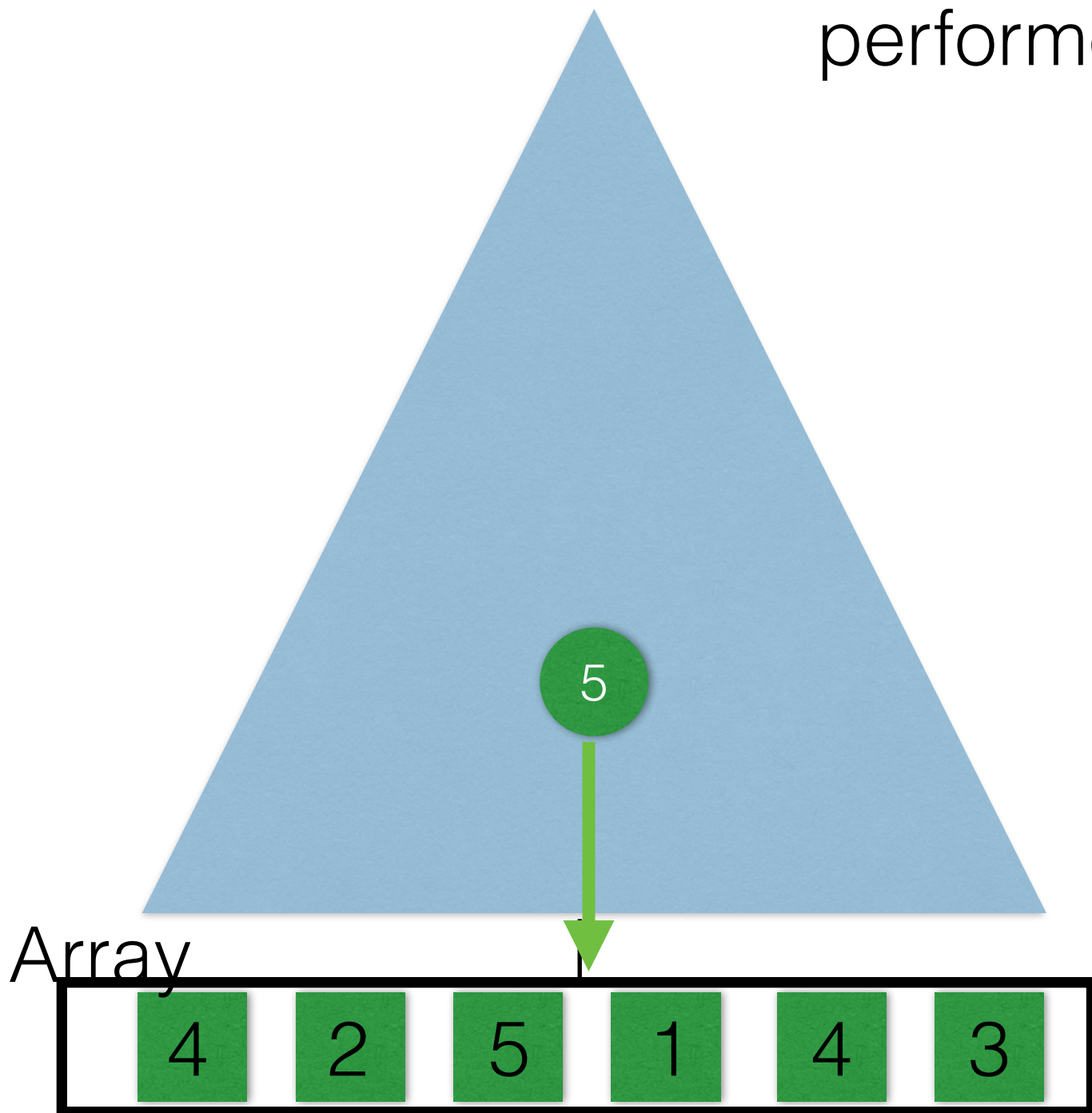
# Reducing the overhead of incremental computation

One memorized computation is performed over an array of data

Improve cache coherence

Reduce incremental overhead

Requires management





# Reducing the overhead of incremental computation

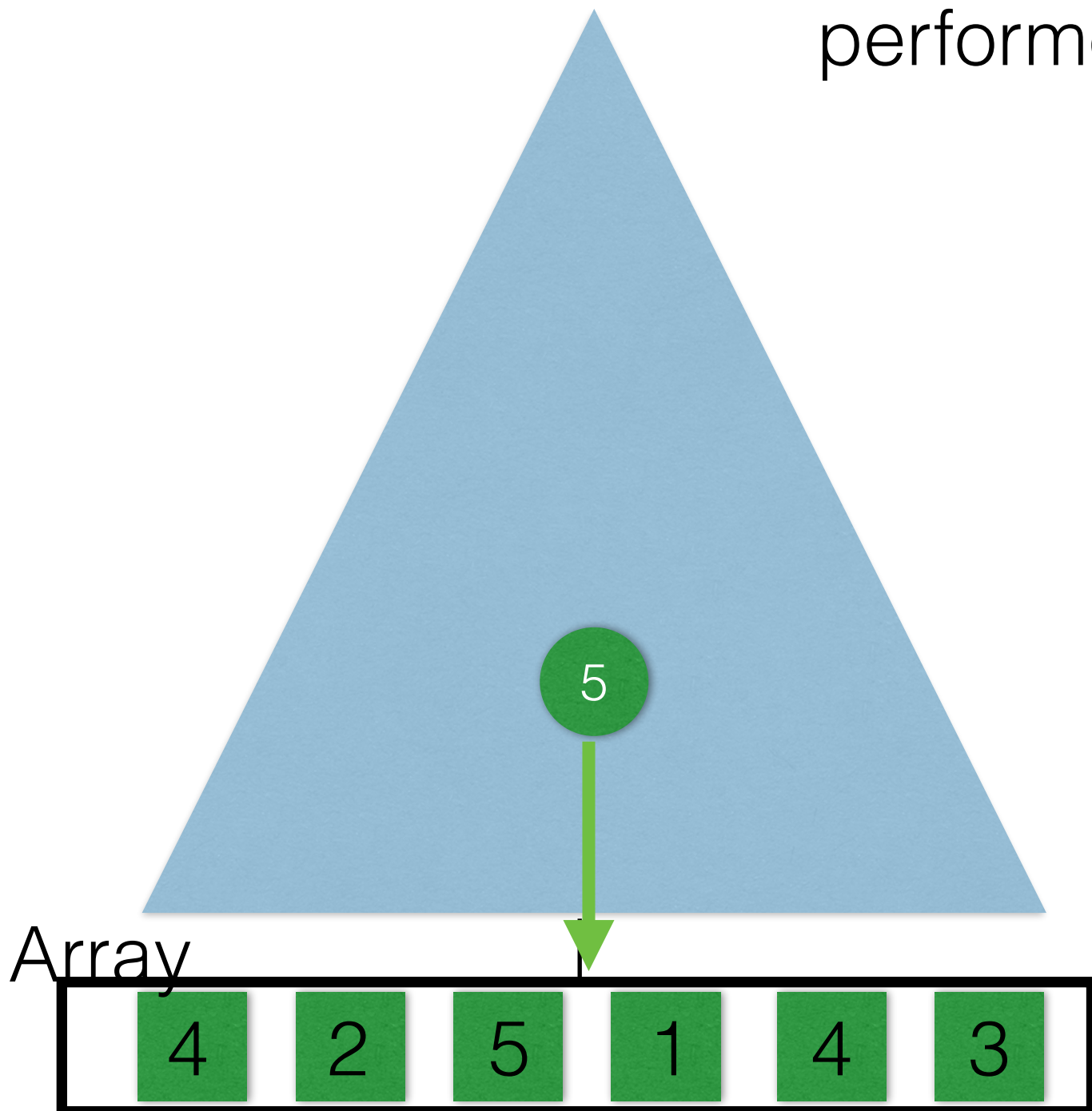
One memorized computation is performed over an array of data

Improve cache coherence

Reduce incremental overhead

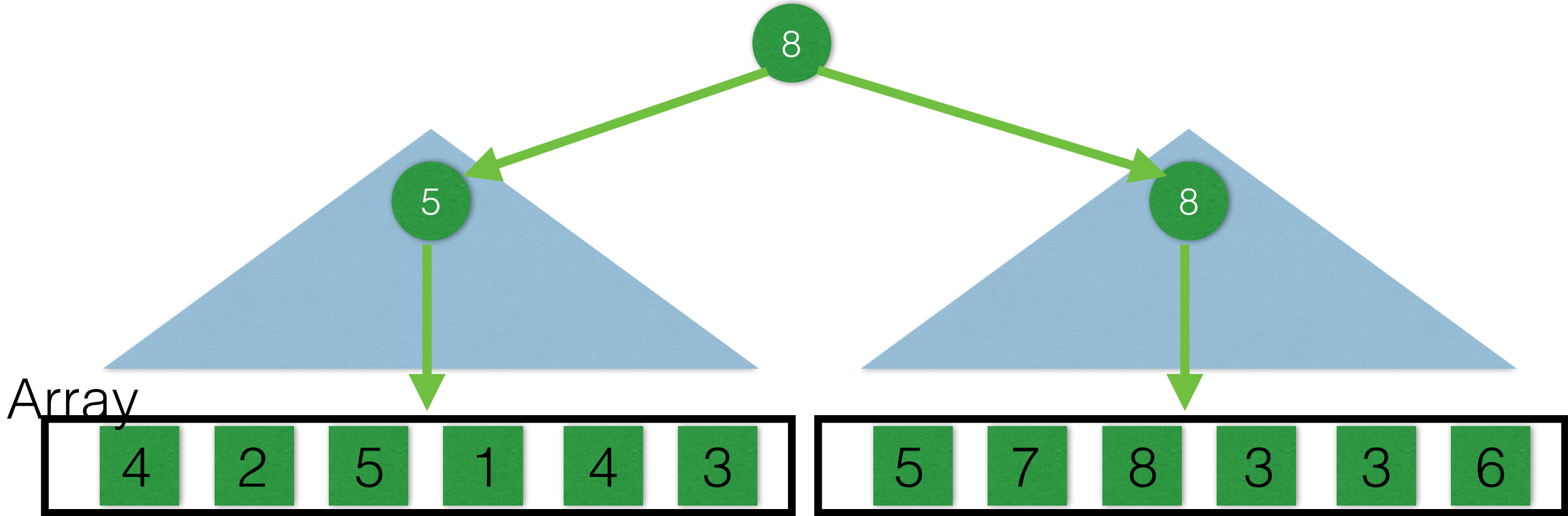
Requires management

We can still do more!



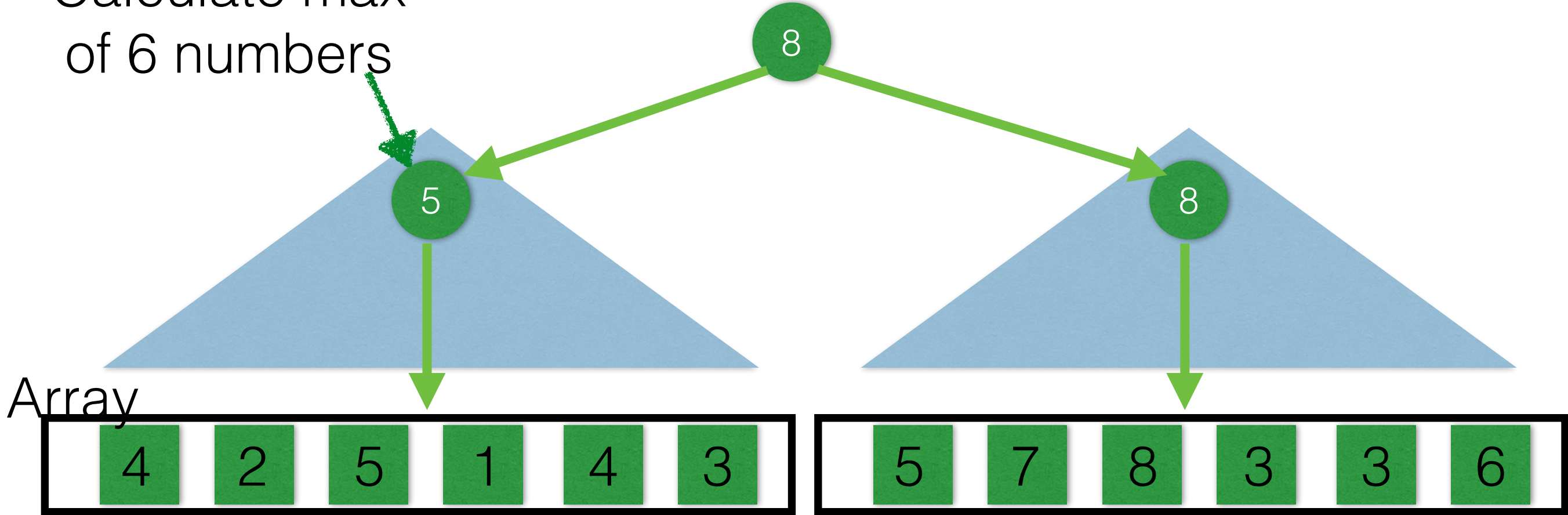


# Reducing the overhead of incremental computation

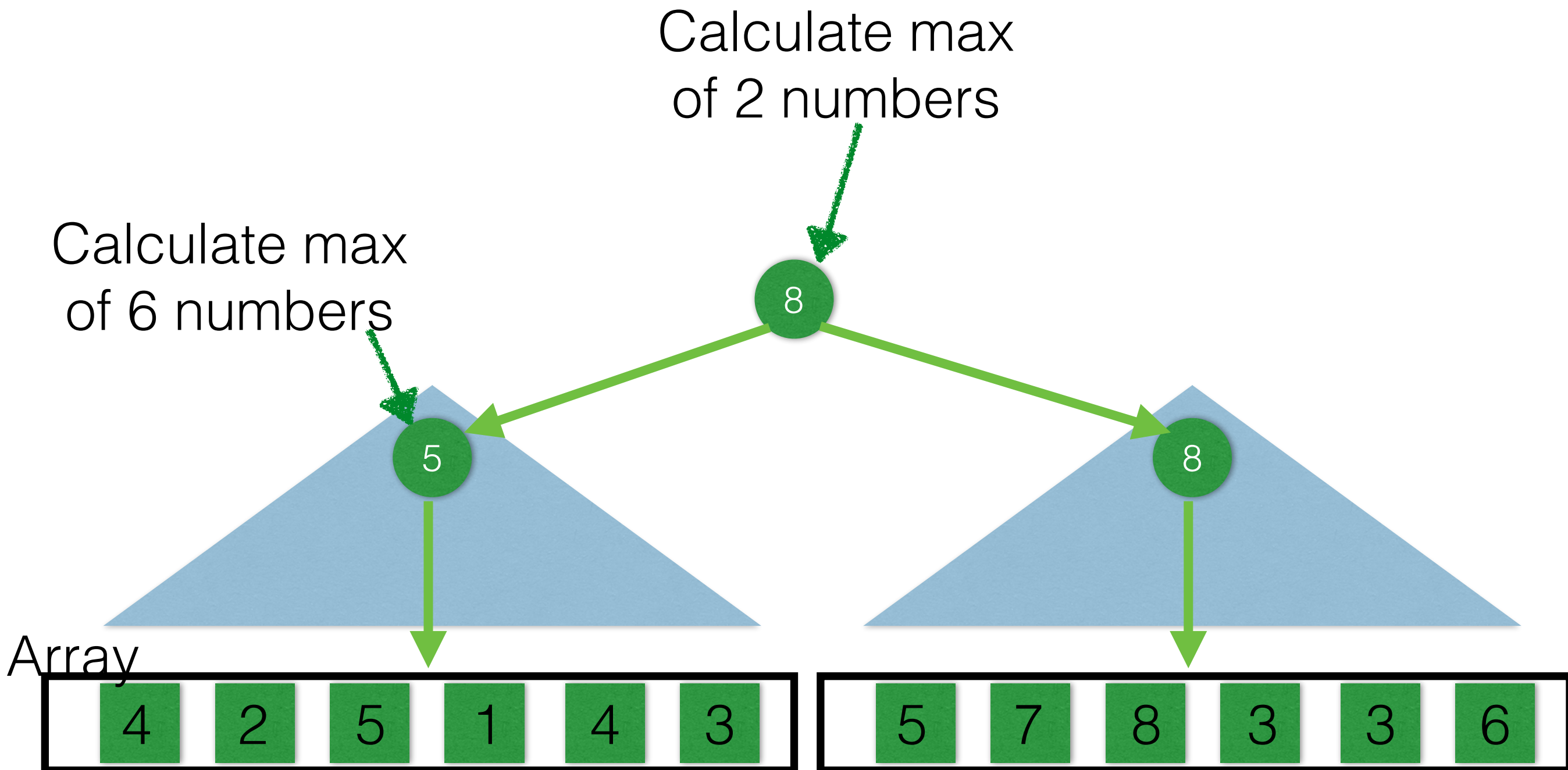


# Reducing the overhead of incremental computation

Calculate max  
of 6 numbers



# Reducing the overhead of incremental computation



# Reducing the overhead of incremental computation

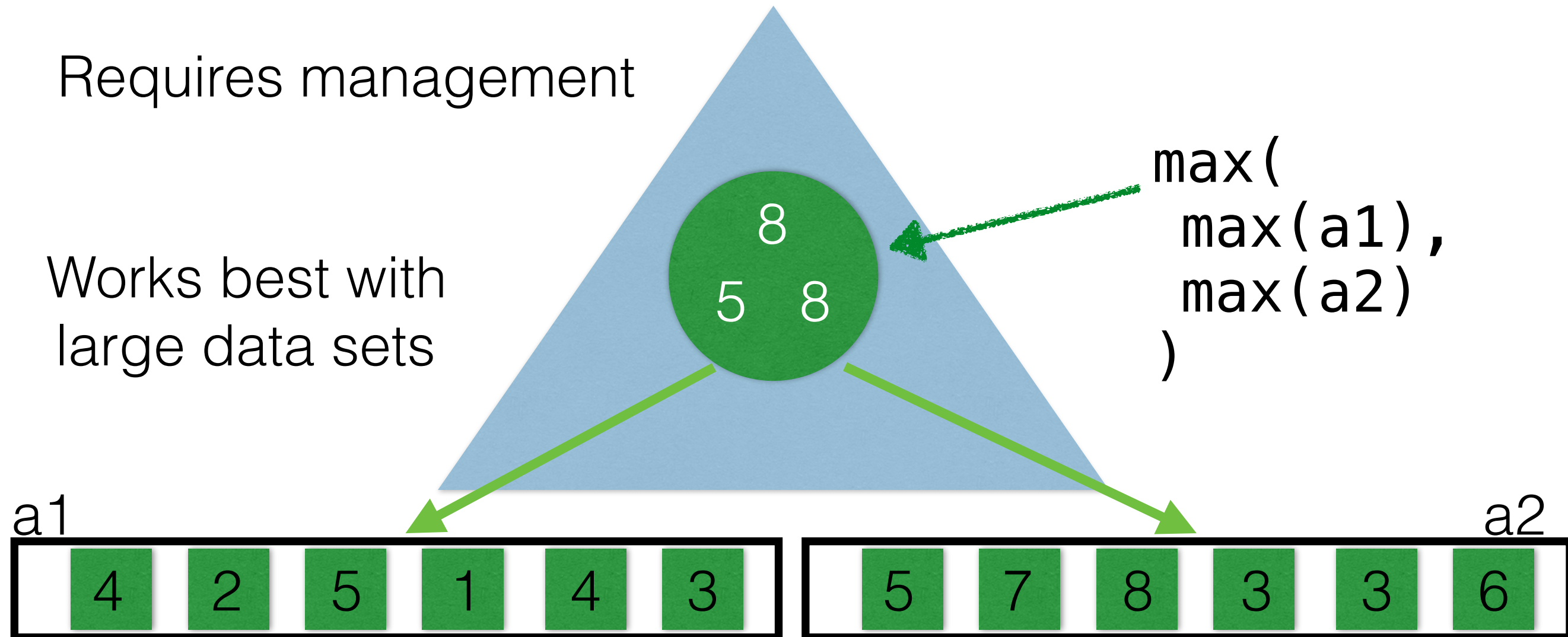
One memorized thunk tracks  
multiple user function calls

Improve cache coherence

Reduce incremental overhead

Requires management

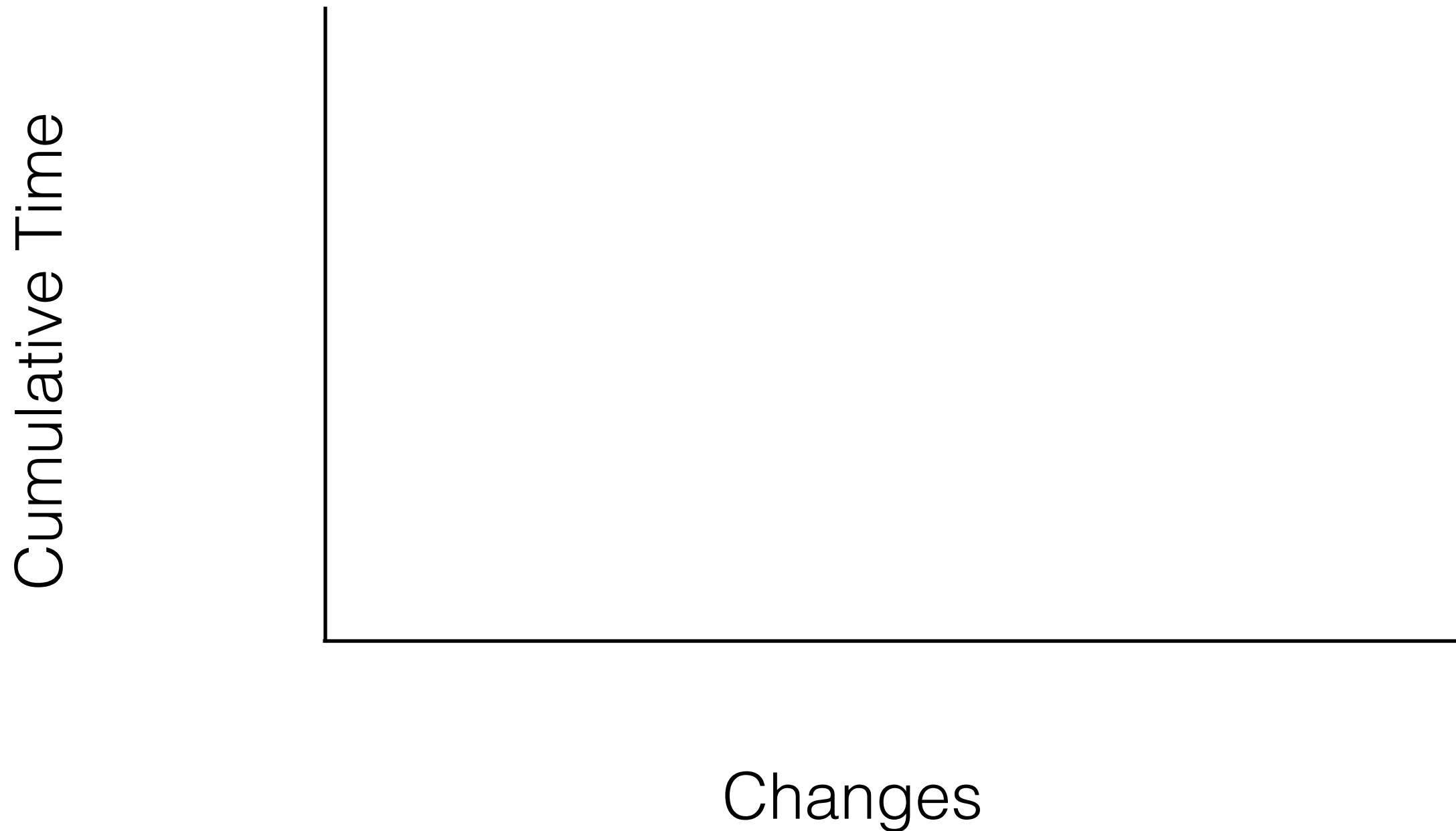
Works best with  
large data sets



# Experimental Evaluation

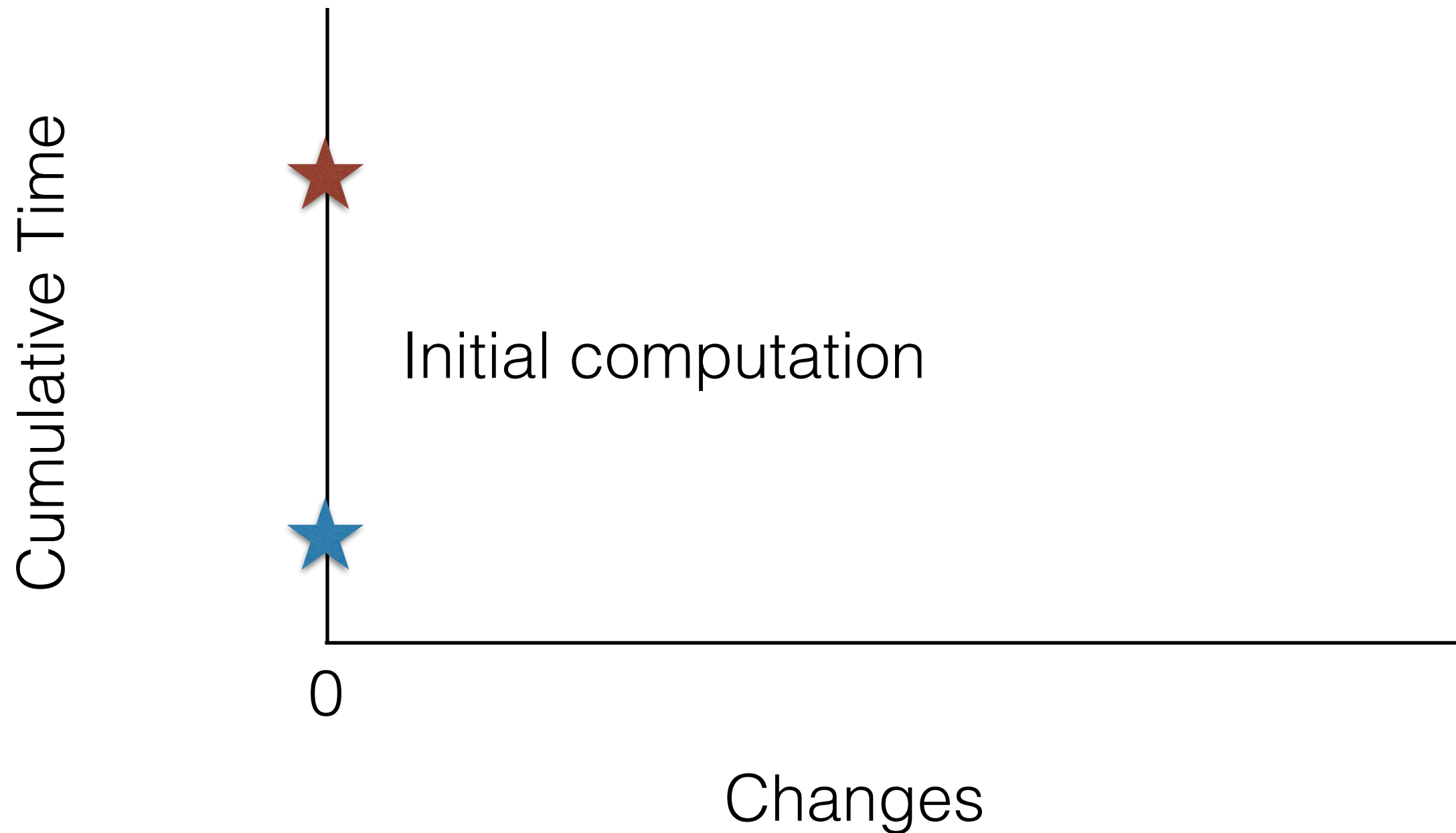
# Experimental Evaluation

Crossover plot



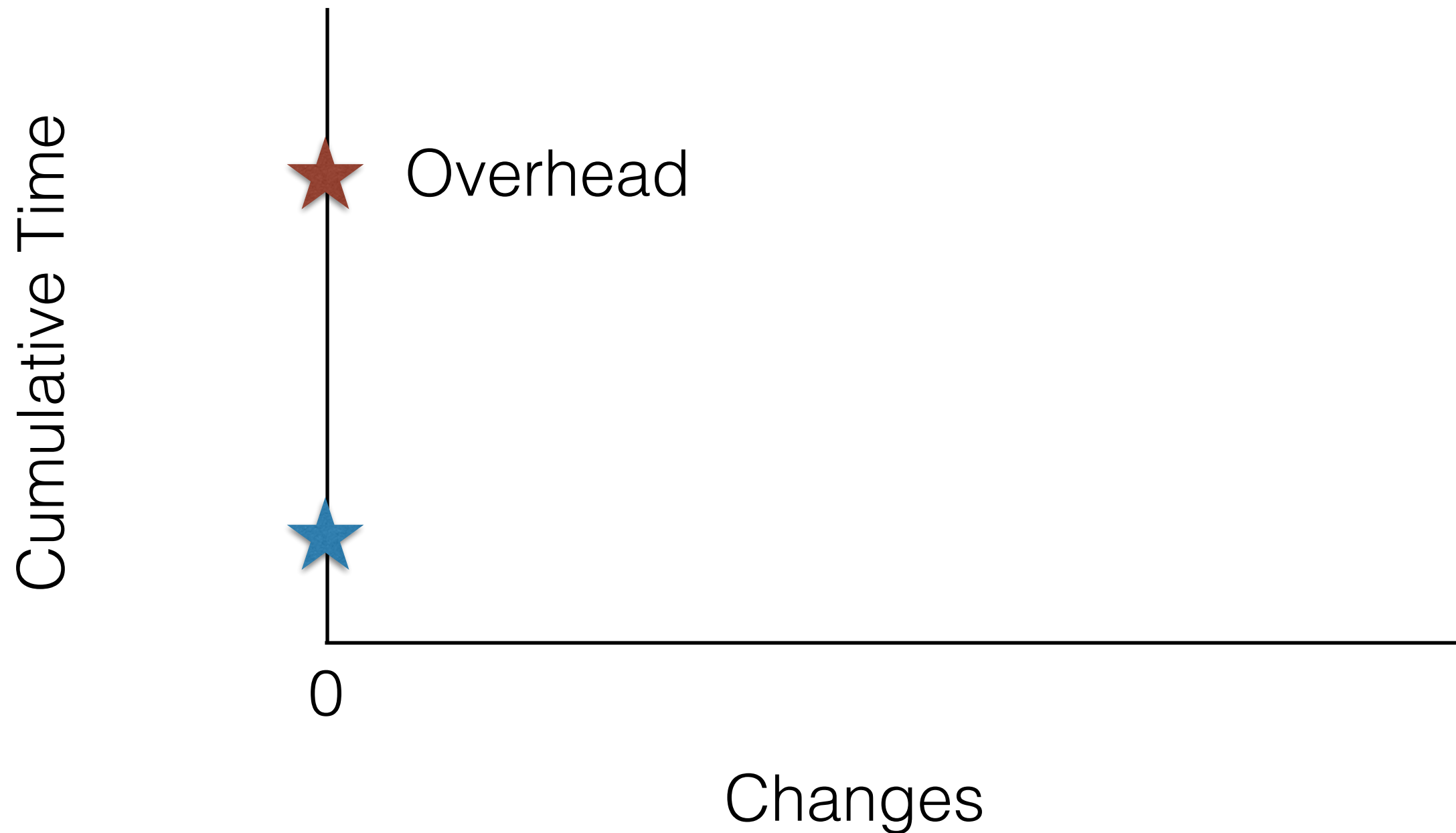
# Experimental Evaluation

Crossover plot



# Experimental Evaluation

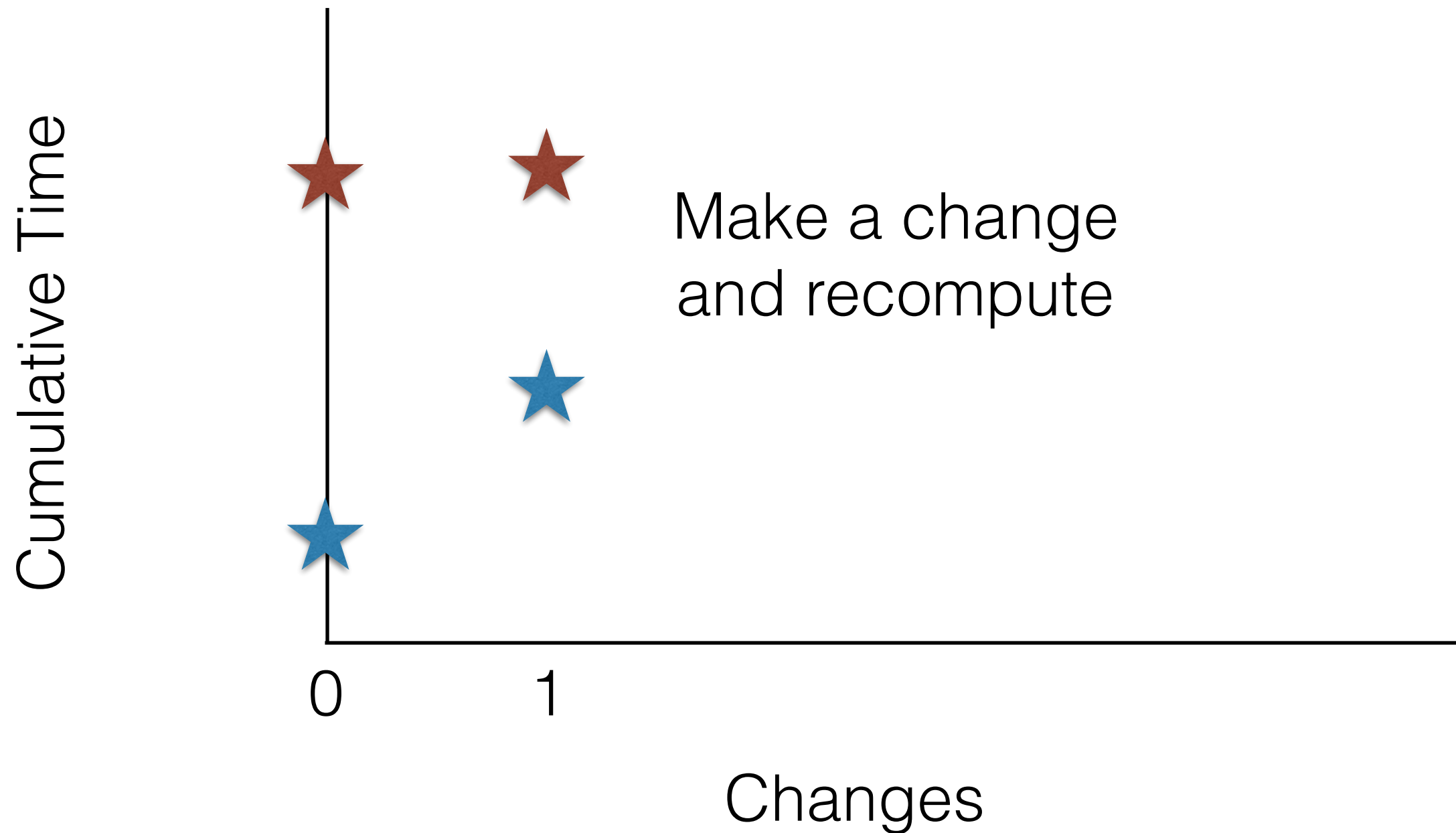
Crossover plot





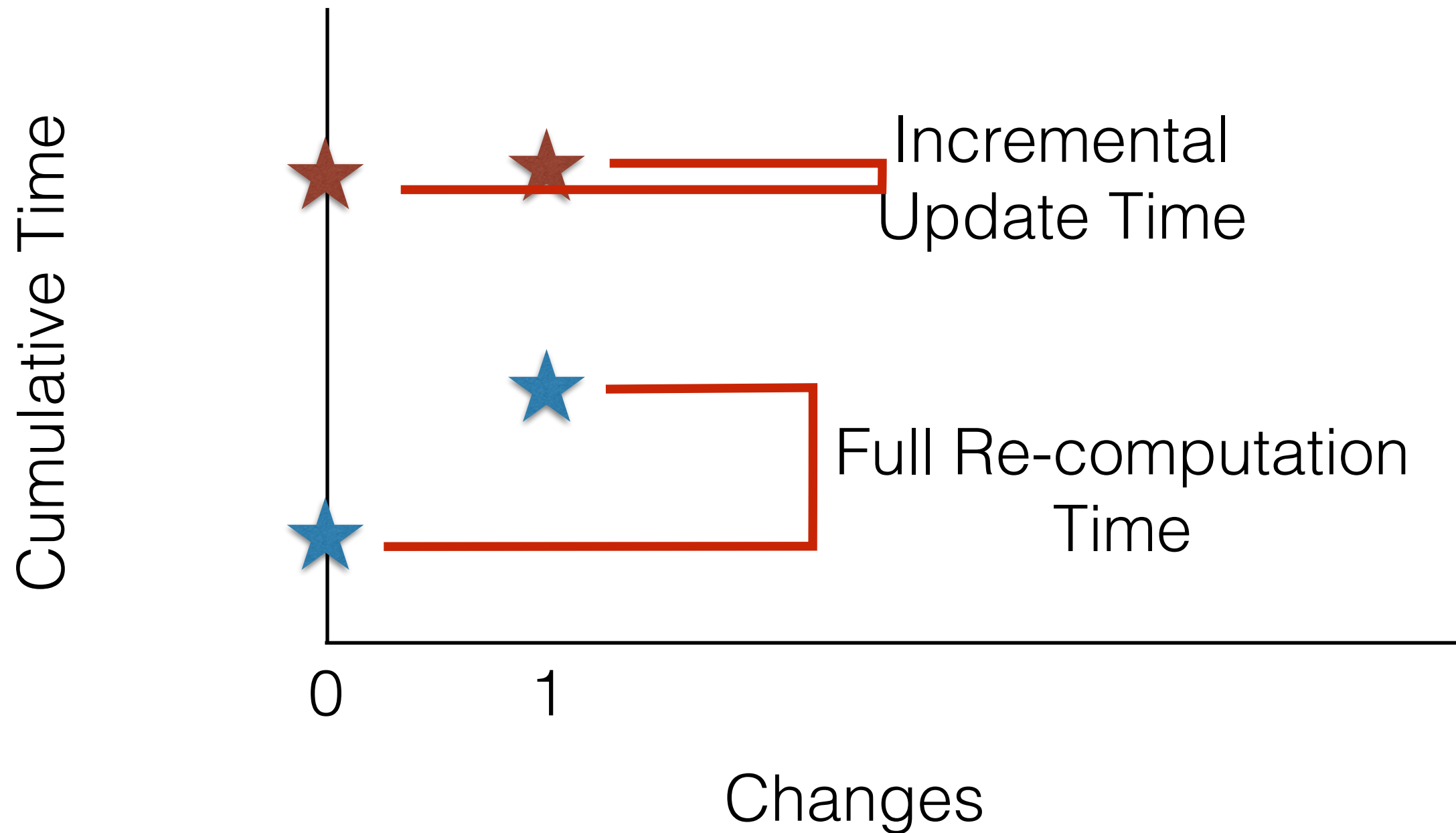
# Experimental Evaluation

Crossover plot



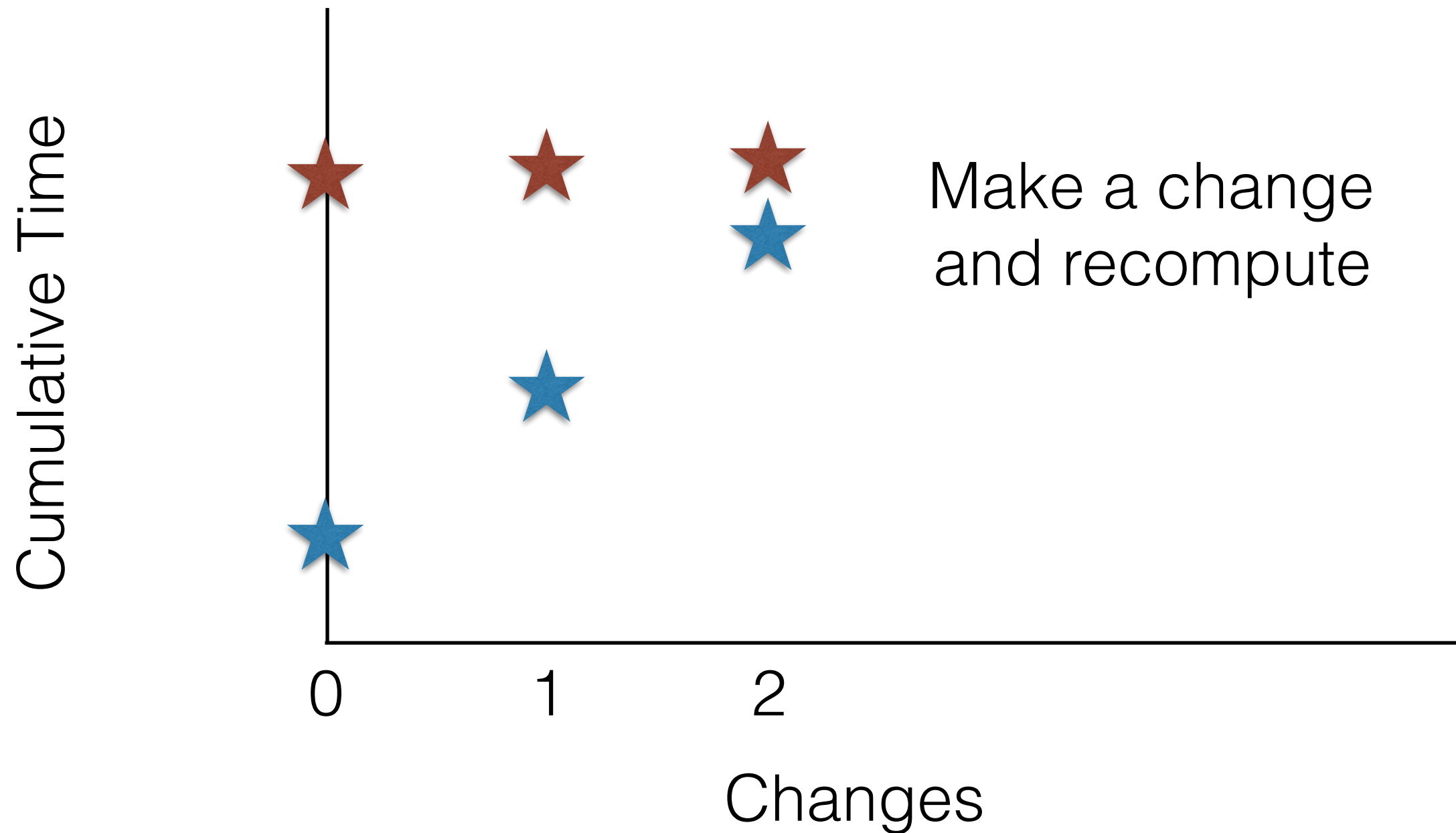
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Crossover plot



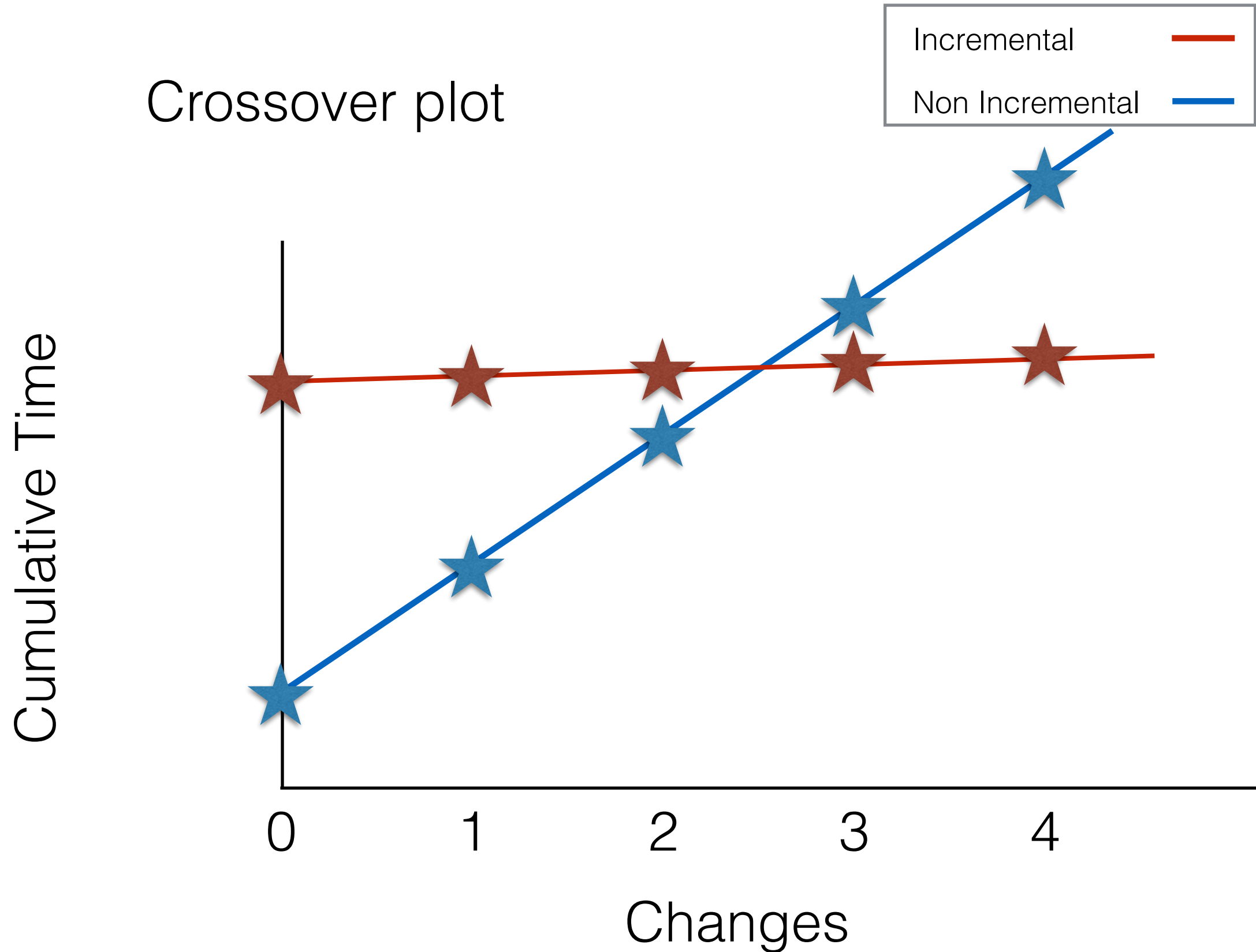
# Experimental Evaluation

Crossover plot



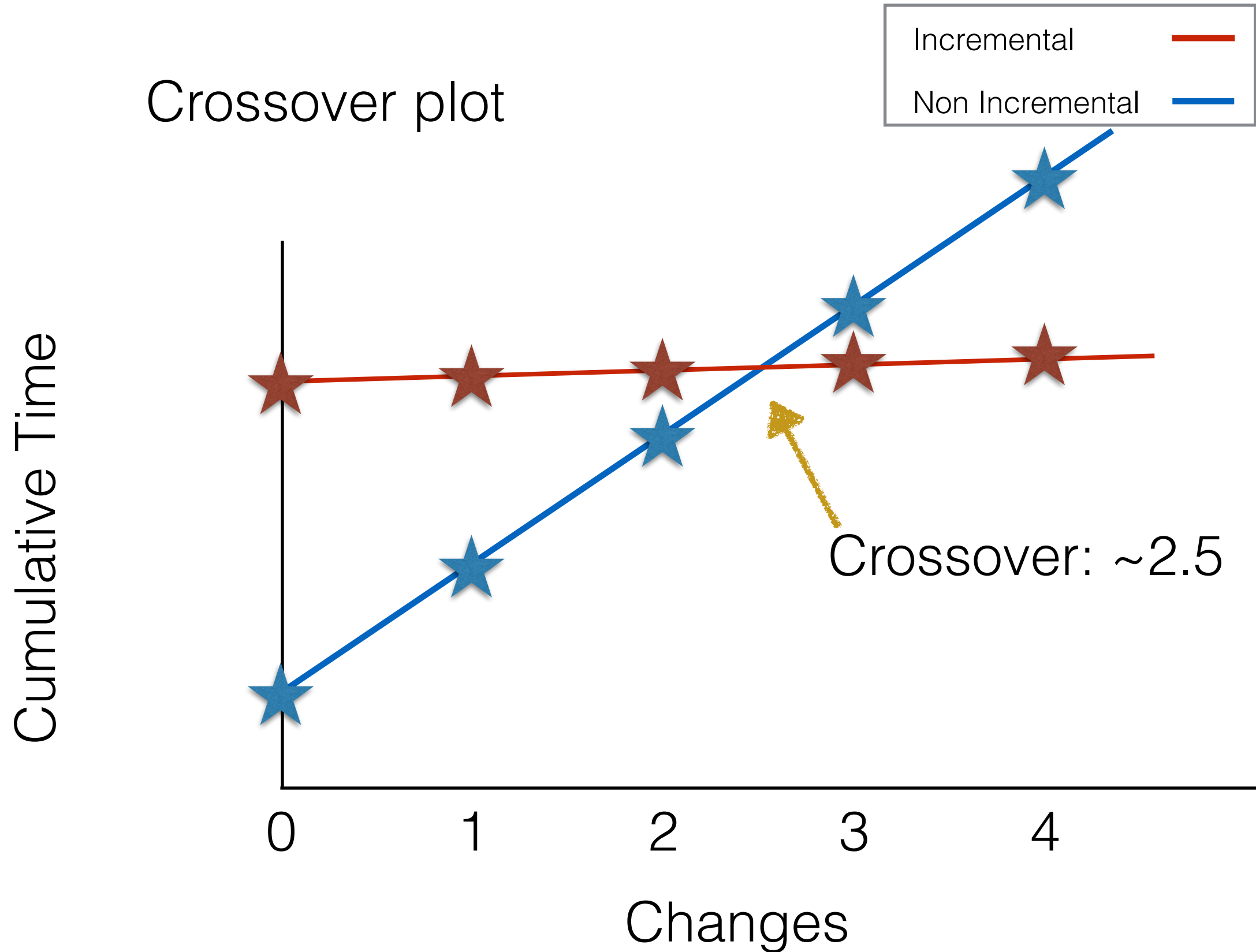
# Experimental Evaluation

Crossover plot



# Experimental Evaluation

Crossover plot



# Experimental Evaluation

Compute max of a collection

Non-Incremental

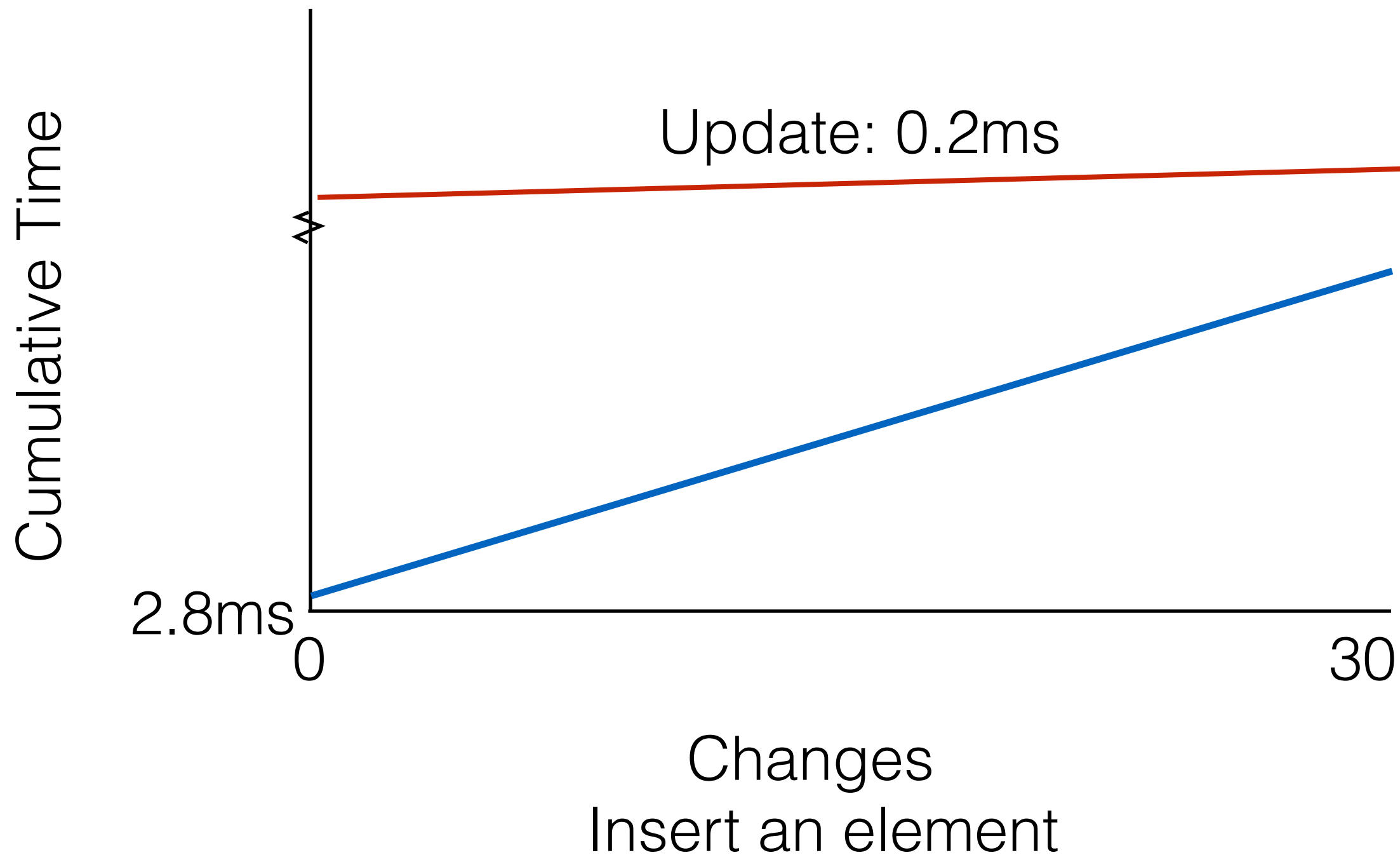
```
inputvec.iter().max()
```

Incremental

```
inputgiraz.fold_up( $\lambda x$ .match x {  
  Leaf(vec) => vec.iter().max(),  
  Bin(m1,m2) => max(m1,m2)  
})
```

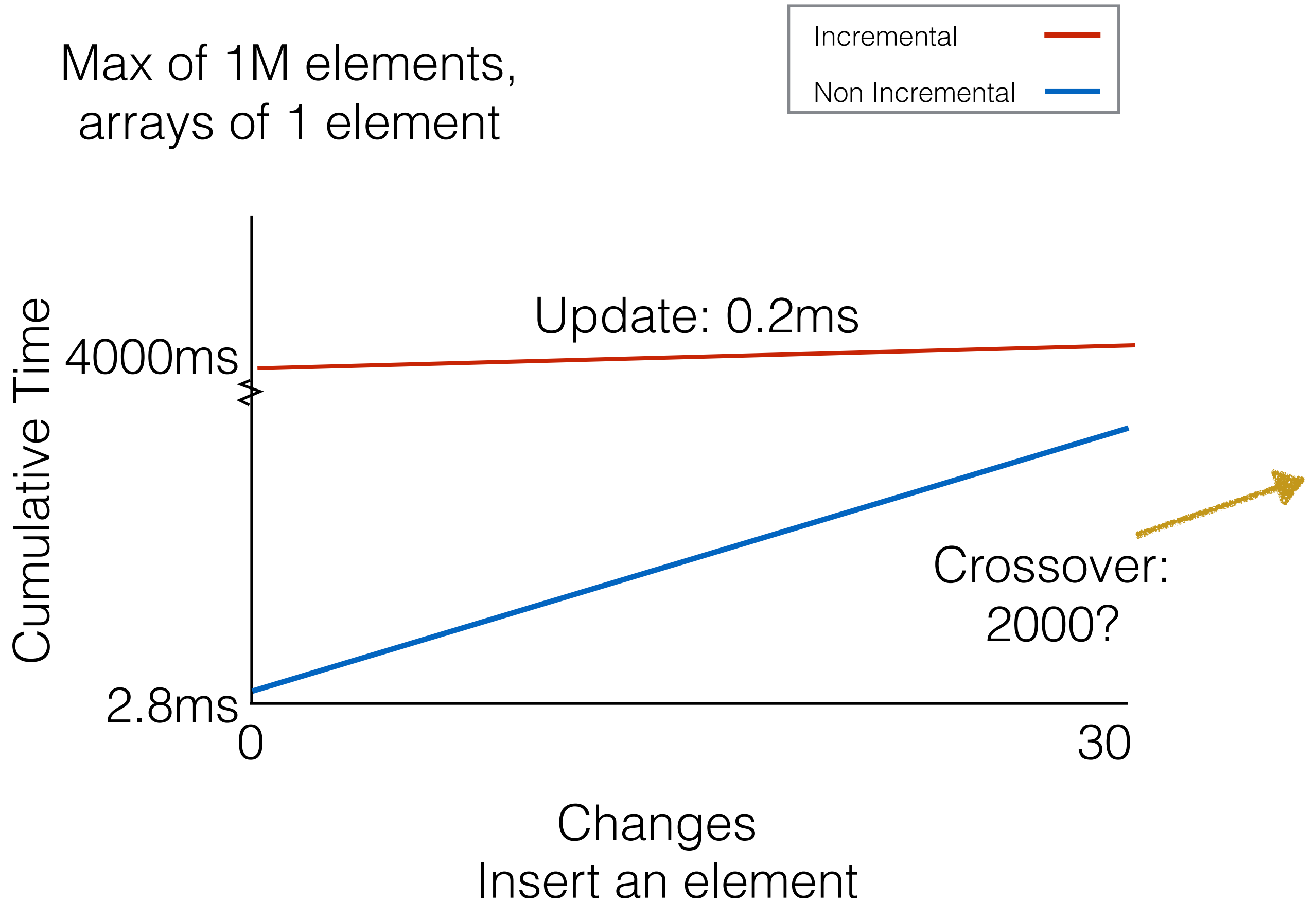
# Experimental Evaluation

Max of 1M elements,  
arrays of 1 element



# Experimental Evaluation

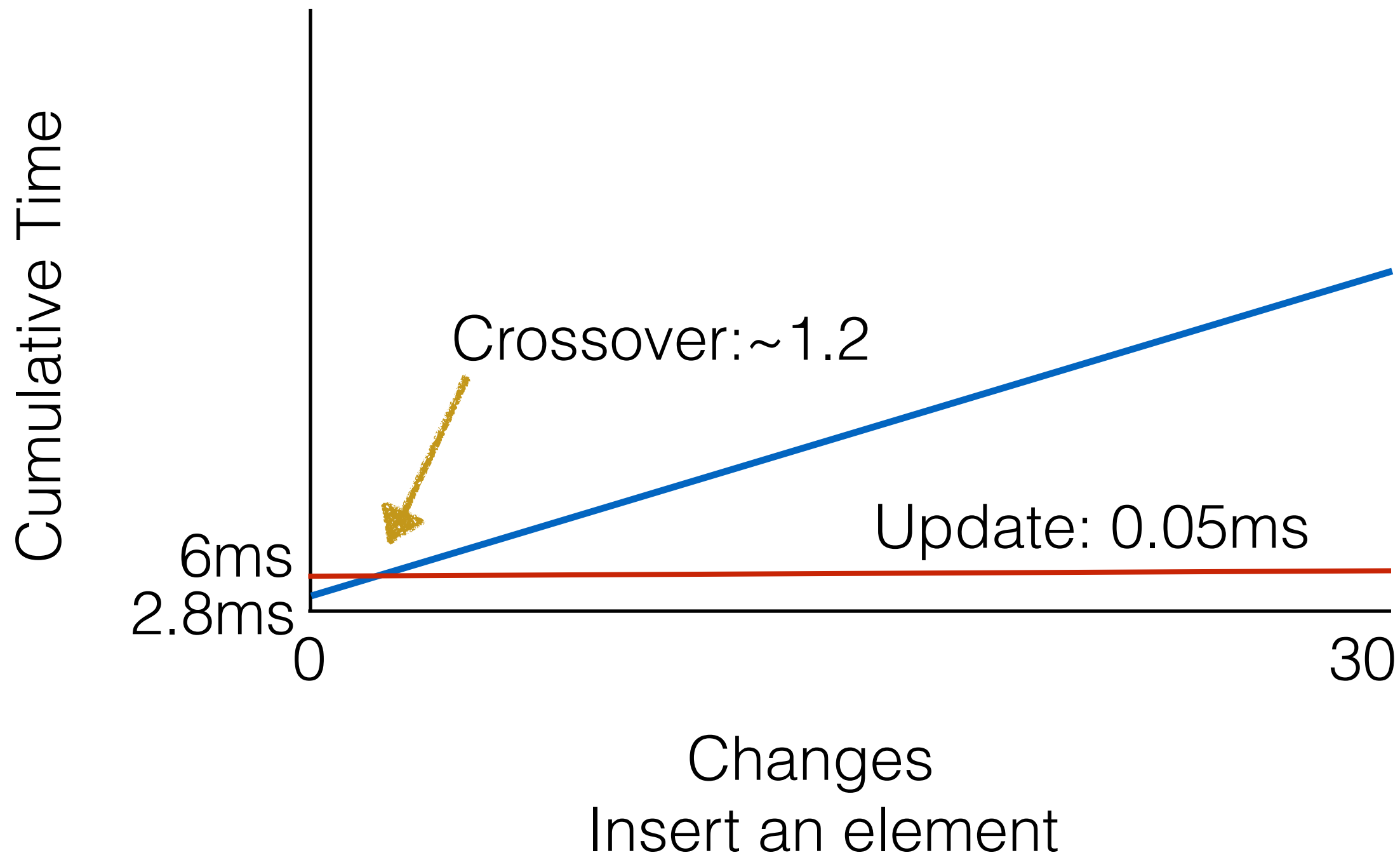
Max of 1M elements,  
arrays of 1 element





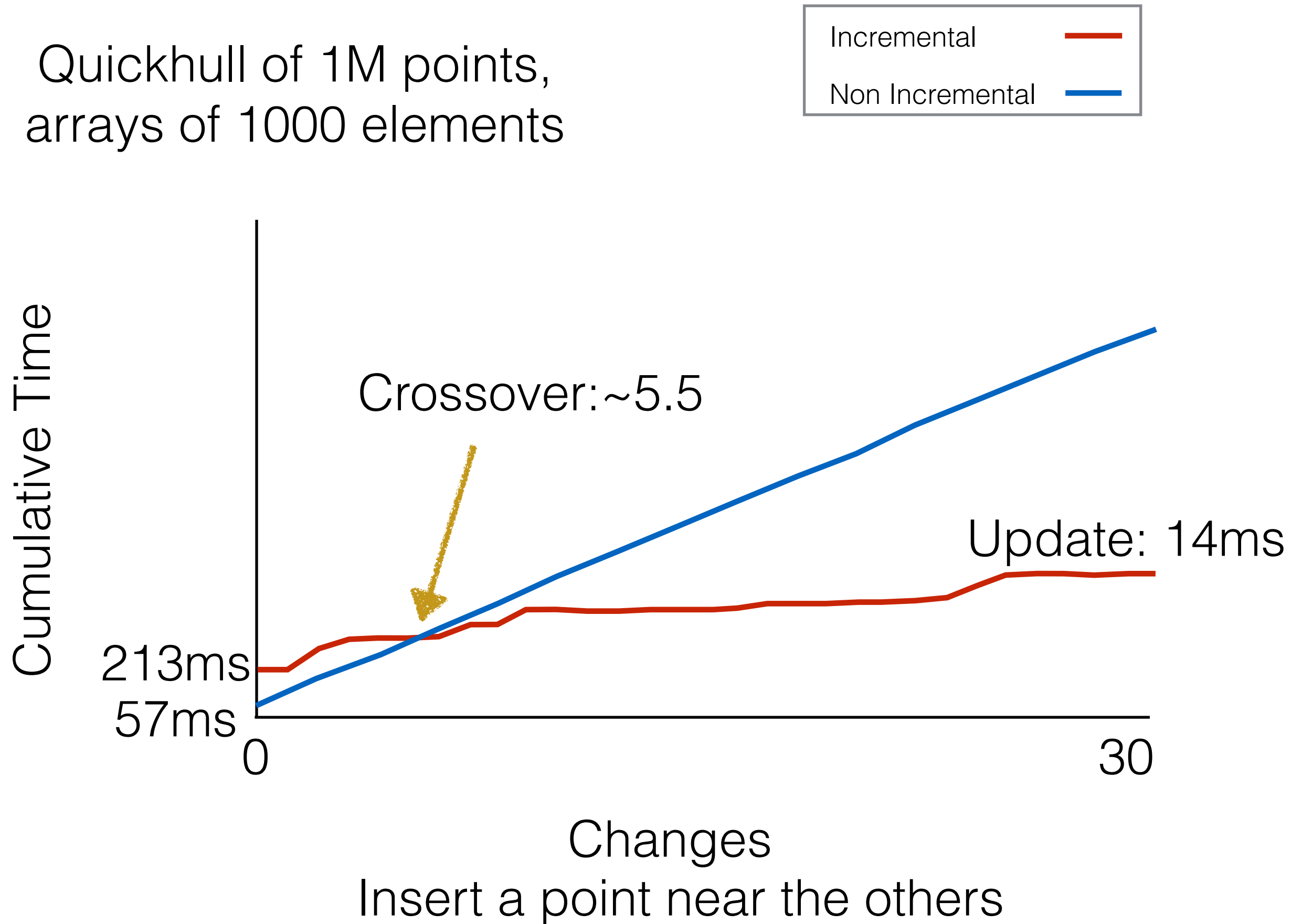
# Experimental Evaluation

Max of 1M elements,  
arrays of 1000 elements



# Experimental Evaluation

Quickhull of 1M points,  
arrays of 1000 elements



All inputs: 1M, gauges 1k, times in ms

	Native initial	inc initial	inc update	crossover	speedup
max	2.84	5.99	0.05	2	57.5
quickhull	56.6	213	13.5	6	4.20
adder	10.3	91.1	0.43	10	23.9
to_string	93.8	95.5	0.21	1	449
reverse	2.01	7.85	0.09	4	22.2

[github.com/cuplv/iodyn.rust](https://github.com/cuplv/iodyn.rust)

cd eval, cargo run --release --example [name] -- [options]

# Summary

Development of an incremental computation library where the user writes non-incremental code

This library is competitive with native rust code

The api allows the user to specify subsequences, which can tune performance to a particular application

Code is available on Github, and can be imported into rust projects through the standard package manager

[www.github.com/cuplv/iodyn.rust](https://www.github.com/cuplv/iodyn.rust)

[kyleheadley.github.io](https://kyleheadley.github.io)